Book Chapter

Multiple Quadratic Polynomial Regression Models and Quality Maps for Tensile Mechanical Properties and Quality Indices of Cast Aluminum Alloys According to Artificial Aging Heat Treatment Condition

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Abstract

To evaluate quality of cast aluminum alloys quantitatively and intuitively, quality index and quality map have been used. Quality index and quality map are to quantitatively evaluate the quality of cast aluminium alloys according to yield strength (*YS*), ultimate tensile strength (*UTS*), elongation to fracture (E_f) and strain energy density (*W*). There are some quality indices such as *Q*, Q_R, Q_C and Q_0. The quality maps are generated to intuitively evaluate the quality level based on the quality indices. These quality indices and quality maps show the quality levels according to the pairs of tensile mechanical properties such as *UTS* and E_f, or *YS* and E_f, or *YS* and *W*. By using these quality maps, it is impossible to directly evaluate the quality levels according to artificial aging heat treatment condition. We develop multiple quadratic polynomial regression models and quality maps for tensile mechanical properties and quality

indices of the cast aluminum alloys according to artificial aging heat treatment condition. The performances of the regression models are evaluated using mean absolute errors, mean relative errors and coefficients of determination. The regression models and quality maps could be widely used to evaluate the quality of the cast aluminium alloys according to aging heat treatment conditions and determine the rational aging heat treatment condition.

Keywords

Cast Aluminum Alloy; Multiple Quadratic Polynomial Regression Model; Quality Index; Quality Map; Tensile Mechanical Property; Artificial Aging Heat Treatment

Introduction

Cast aluminum alloys are widely used in modern industry because of good corrosion resistance, high level of mechanical properties such as ultimate tensile strength, yield strength and elongation, and a good castability [1]. Quality level of cast aluminum alloys are quantified using quality index. The quality index is a measure to evaluate the quality of cast aluminum alloys based on tensile mechanical properties such as yield strength (YS), ultimate tensile strength (UTS), elongation to fracture (E_f) and strain energy density (W). It enables the material designers and engineers to select the reasonable materials with high quality and it is considered to be a key factor for selecting an alloy for a particular engineering application [2,3]. The quality index allows to compare different castings, which may have received different heat treatment conditions, or whose chemical compositions are different [4].

There are some popular quality indices such as Q, Q_R, Q_C and Q_D. The calculation formulas are as follows [2,5]:

$$Q = UTS + d \cdot \log_{10}(E_f) \quad (1)\ [6]$$

$$Q_R = YS + m \cdot E_f \quad (2)\ [7]$$

$$Q_C = UTS + 0.4 \cdot YS \cdot [E/(a \cdot YS)]^n \cdot \log_{10}(E_f) \qquad (3)\ [8]$$

$$Q_D = K_D \cdot Q_0,\ Q_0 = YS + 10 \cdot W \qquad (4)\ [9]$$

In the above equations, *UTS* is the ultimate tensile strength, E_f is the elongation at fracture, *YS* is the yield strength and *W* is the strain energy density. *d* is an empirical coefficient chosen such as to make *Q* practically independent of the aging condition and it takes the value of 150MPa in the original work. *m* is a material constant expressed in MPa. The value of *m* varies according to considered alloys: 7.5–13MPa for the Al–Cu alloys, 50MPa for the Al–Si–Mg alloys, 7.5MPa for A206 alloys, 13MPa for A201 alloys, 50MPa for A356 and A357 alloys, and 40MPa for Al-7%Si-Mg Alloys. *E* is the Youngs modulus and *n* the strain hardening exponent and *a* a scale factor of order.5 In Eq. 4, Q_0 takes into account the average tensile mechanical properties of the materials and K_D takes into account the scatter in the mechanical properties. Using the quality index Q_D and by neglecting the scatter in mechanical properties, i.e, by assuming K_D= 1, the quality level of an alloy can be characterized by Q_0= YS+10·W [5,9].

The quality index *Q* is more sensitive to tensile strength than to tensile ductility variation and the quality indices Q_C and *Q* follow absolutely the almost same alloy quality evaluation. The quality index Q_R is strongly governed by the ductility property of the material and more sensitive to ductility, and it favors the ductile materials more than Q_D [10]. The quality index Q_D is well balanced over strength and ductility of the materials [11]. It evaluates the material's potential to offer combinations of tensile strength, ductility and toughness. The quality index Q_D evaluates the material quality on the basis of a balance between the material properties' yield strength *YS* and strain energy density *W* [12].

The concept of quality map for supporting material selection was introduced since several decades and exploited in a series of applications [6,8]. Ammar et al. [3] considered the quality of Al-Si casting alloys to be a key factor in selecting an alloy for a particular engineering application. They reviewed the development of the quality index concept and quality chart in relation to their application to Al–Si casting alloys. They reviewed different theories pertaining to the concept of the quality index *Q*, its calculation, and its use in the construction of quality charts. The quality charts are used to evaluate the mechanical properties and the quality of the castings under the influence of various metallurgical parameters, and the quality chart concept has thus already been used as a selection tool for engineering materials. The quality charts are generated for use as a simple method of evaluating, selecting, and also predicting the most appropriate metallurgical conditions which may be applied to the castings so as to obtain the best possible compromise between tensile properties and casting quality. Alexopoulos and Pantelakis9 plotted a diagram of the ultimate tensile strength vs the logarithm of the elongation to fracture. In the diagram, $Q = UTS + d \cdot \log_{10}(E_f)$ and $YS = a \cdot UTS - b \cdot \log_{10}(E_f) + c$ represent the sets of parallel lines called iso-quality index and iso-yield strength lines, respectively; they fit the experimentally obtained *Q* and YS values resulting from variations in chemical composition, solidification conditions, and heat treatment of Al-Si-Mg aluminum alloys with a good approximation. Hence, the preceding diagram called quality index chart provides a very useful tool to reduce the experimental effort for developing or optimizing Al-Si-Mg cast alloys essentially. Alexopoulos10 investigated the use of quality indices of cast aluminum alloys to support material selection by means of generation of quality maps. They generated the quality maps of cast aluminum alloys from different series, the same series and minor variations in chemical composition, and the same series and variations in heat treatment. The quality maps enable to support the material selection based on all proposed quality indices, devised for the evaluation of cast aluminum alloys to be used in aircraft applications. Alyaldin et al. [13] performed the study on the 354 (Al–9wt%Si–1.8wt%Cu–0.5wt%Mg)-based alloy to which measured amounts of Zr, Ni, Mn and Sc were added. They used the quality index *Q* to calculate the quality index values of the aluminum alloy castings and plot the iso-Q lines and the iso-YS lines, respectively. They carried out the analysis of the tensile data using quality charts and color contour maps. Pantelakis et al. [14] generated the quality map of the investigated AM cast magnesium alloys with variations in the Al content, the quality map of QE22A cast magnesium alloy for different solid solution heat treatment temperatures and the quality map of QE22A cast magnesium alloy for the different artificial aging heat treatment conditions on the basis of the quality index Q_0. They compared the mechanical performances of cast magnesium and aluminum alloys by means of the quality maps. The quality maps support the design engineer to easily select the process conditions of QE22A that optimize the material's mechanical performance.

The previous quality indices and quality maps show the quality levels of the cast aluminum alloys according to only the pairs of tensile mechanical properties such as *UTS* and E_f, or *YS* and E_f, or *YS* and *W*. The research gaps of the previous studies are that it is impossible to show the tensile mechanical properties and quality levels according to artificial aging heat treatment conditions, directly.

If we develop the relation models and quality maps to evaluate the tensile mechanical properties and quality indices of the cast aluminum alloys according to artificial aging heat treatment condition, it is possible to grasp the influences of the artificial aging heat treatment conditions to their quality levels and decide the reasonable artificial aging heat treatment conditions, directly. It is great convenience and effective to evaluate the quality level of the cast aluminum alloy according to the artificial aging heat treatment conditions in practice.

For this purpose, we develop the regression models and quality maps for tensile mechanical properties and quality indices according to artificial aging heat treatment condition in this paper.

Commonly, to model the relationship between the response variable *y* and several independent variables (factors), multiple linear regression model, multiple polynomial regression model and artificial neural network (ANN) model are used. The multiple linear regression model is simple and easy to use in practice. When the performance is not satisfied, the multiple polynomial regression model such as multiple quadratic or cubic polynomial regression model is used. When the performance is not also satisfied, the ANN model such as BPNN model is used. The ANN model has an excellent learning capacity. However, it needs large amount of data for developing the model and the performance is highly dependent on having sufficient data collections.

To develop the relationship model with high performance, various research works were conducted by using artificial intelligent-based methodologies such as novel metaheuristic-based machine learning method, interpretable data-intelligence model, data driven models and soft computing techniques [15-18].

In this paper, the regression models for the tensile mechanical properties and quality indices of the cast aluminum alloy A357 according to artificial aging heat treatment condition are developed using multiple quadratic polynomial regression models and the quality maps are plotted based on the regression models.

Methods

Method to develop Multiple Quadratic Polynomial Regression Model

The general form of multiple linear regression model of a dependent variable (response) y according to p independent variables (factors) x_1, x_2, .., x_p can be expressed as follows [19,20]:

$$\begin{cases} y_1 = \beta_0 + \beta_1 x_{11} + \cdots + \beta_p x_{1p} + e_1 \\ y_2 = \beta_0 + \beta_1 x_{21} + \cdots + \beta_p x_{2p} + e_2 \\ \cdots \\ y_n = \beta_0 + \beta_1 x_{n1} + \cdots + \beta_p x_{np} + e_n \end{cases}, \quad (5)$$

i.e.

$$y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_p x_{ip} + e_i \text{, for } i=1, 2, .., n, \quad (6)$$

where y_i is the value of the response variable y for the ith case, x_{ij} is the value of the jth independent variable x_j for the ith case, β_0 is the y-intercept of the regression surface (think multidimensionality), each β_j is the slope of the regression surface with respect to variable x_j and e_i is the random error component for the ith case. In Eq. 5, we have n observations and p factors ($n > p+1$).

Eq. 5 can be rewritten in matrix notation as follows:

$$y = X \cdot \boldsymbol{\beta} + \boldsymbol{e}, \qquad (7)$$

where response vector $\boldsymbol{y}$ and error vector $\boldsymbol{e}$ are the column vectors of length n, vector of parameters $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2,.., \beta_p)'$ is a column vector of length p+1 and design matrix X is n by p+1 matrix (with its first column having all elements equal to 1, the second column being filled by the observed values of x_1, the third column being filled by the observed values of x_2, etc.).

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ \cdots \\ y_n \end{pmatrix}, \quad X = \begin{pmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1p} \\ 1 & x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{np} \end{pmatrix}, \quad \boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \cdots \\ \beta_p \end{pmatrix}, \quad \boldsymbol{e} = \begin{pmatrix} e_1 \\ e_2 \\ \cdots \\ e_n \end{pmatrix}. \qquad (8)$$

The vector of regression parameters $\boldsymbol{\beta}$ could be estimated using the least square method as follows:

$$\hat{\boldsymbol{\beta}} = (X^T X)^{-1} X^T Y, \qquad (9)$$

where $\hat{\boldsymbol{\beta}}$ is an unbiased estimator of $\boldsymbol{\beta}$.

In practical applied work, the quadratic terms such as x_2^1 and x_2^2 are often included to model a curved relationship between y and several independent variables (factors).

The general form of the multiple quadratic polynomial regression model is as follows:

$$y = \beta_0 + \sum_{j=1}^{p} \beta_j x_j + \sum_{j=1}^{p} \sum_{k=j}^{p} \beta_{j,k} x_j x_k$$
$$= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p + \beta_{1,1} x_1^2 + \beta_{1,2} x_1 x_2 + \cdots + \beta_{1,p} x_1 x_p + \quad (10)$$
$$\beta_{2,2} x_2^2 + \beta_{2,3} x_2 x_3 + \cdots + \beta_{2,p} x_2 x_p + \cdots + \beta_{p\text{-}1,p\text{-}1} x_{p\text{-}1}^2 + \beta_{p-1,p} x_{p-1} x_p + \beta_{p,p} x_p^2,$$

where β_0 is a constant, and β_j, $\beta_{j,j}$ and $\beta_{j,k}$ are the linear, pure quadradic and interaction coefficients, respectively.

Let

$X_1 = x_1, X_1 = x_1, \cdots, X_p = x_p, X_{p+1} = x_1^2, X_{p+2} = x_1 x_2, X_{p+3} = x_1 x_3, \cdots, X_{m-1} = x_{p-1} x_p, X_m = x_p^2$
.

The multiple quadratic regression model can be represented as the following multiple linear regression model:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \beta_{1,1} X_{p+1} + \beta_{1,2} X_{p+2} + \cdots + \beta_{p,p} X_m . \quad (11)$$

Therefore, the multiple quadratic regression model could be considered as a particular case of multiple linear regression model.

The least squared estimator for the multiple quadratic regression model are as follows:

$$\hat{\boldsymbol{\beta}} = \left(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \cdots, \hat{\beta}_{p,p}\right)^T = \left(X^T X\right)^{-1} X^T \boldsymbol{y}, \quad (12)$$

Where

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ \cdots \\ y_n \end{pmatrix}, \quad X = \begin{pmatrix} 1 & X_{11} & X_{12} & \cdots & X_{1m} \\ 1 & X_{21} & X_{22} & \cdots & X_{2m} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & X_{n1} & X_{n2} & \cdots & X_{nm} \end{pmatrix} = \quad (13)$$
$$\begin{pmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1p} & x_{11}^2 & x_{11}x_{12} & \cdots & x_{11}x_{1p} & x_{12}^2 & x_{12}x_{13} & \cdots & x_{12}x_{1p} & \cdots & x_{1,p-1}x_{1p} & x_{1p}^2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & \vdots \\ 1 & x_{i1} & x_{i2} & \cdots & x_{ip} & x_{i1}^2 & x_{i1}x_{i2} & \cdots & x_{i1}x_{ip} & x_{i2}^2 & x_{i2}x_{i3} & \cdots & x_{i2}x_{ip} & \cdots & x_{i,p-1}x_{ip} & x_{ip}^2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{np} & x_{n1}^2 & x_{n1}x_{n2} & \cdots & x_{n1}x_{np} & x_{n2}^2 & x_{n2}x_{n3} & \cdots & x_{n2}x_{np} & \cdots & x_{n,p-1}x_{np} & x_{np}^2 \end{pmatrix}.$$

As a result, the multiple quadratic regression model is represented as follows:

$$y = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \cdots + \hat{\beta}_p x_p + \hat{\beta}_{1,1} x_1^2 + \hat{\beta}_{1,2} x_1 x_2 + \cdots + \hat{\beta}_{1,p} x_1 x_p +$$

$$\hat{\beta}_{2,2} x_2^2 + \hat{\beta}_{2,3} x_2 x_3 + \cdots + \hat{\beta}_{2,p} x_2 x_p + \cdots + \hat{\beta}_{p\text{-}1,p\text{-}1} x_{p\text{-}1}^2 + \hat{\beta}_{p-1,p} x_{p-1} x_p + \hat{\beta}_{p,p} x_p^2 \quad (14)$$

In order to evaluate the performance of the multiple regression model, the mean absolute error, mean relative error and R-squared R^2 (coefficient of determination).

The mean absolute error (MAE) and mean relative error (MRE) are calculated as follows:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|y_i - \hat{y}_i\right|,$$

$$MRE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \times 100 \ \ (\%), \quad (15)$$

where y_i is observed value and $\hat{y}_i$ is the fitted value of the response variable y for the ith case.

The coefficient of determination (R^2) of the multiple regression model is calculated as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}, \quad (16)$$

where $\bar{y}$ is the mean value of response variable y. R^2 measures the percentage of variation in the response variable y explained by the explanatory variable x_1, x_2, .., x_p. Thus, it is an important measure of how well the regression model fits the data.

For external validation of the multiple regression model, cross-validation (CV) method is used in this paper. The CV method was proposed to evaluate the predictive validity of a linear regression model [21]. One data sample from among the training data is set aside as validation data, and the remaining N-1 data samples are used as construction data set to build the model. The quality of the model is assessed with the validation data. This procedure is repeated N times to get an average cross-validated indices such as MAE, MRE and R^2. The average cross-validated indices are used to validate the model.

If *MRE* is less than 10%, it is interpreted as excellent accurate forecasting, between 10–20% good forecasting, between 20–50% acceptable forecasting and over 50% inaccurate forecasting.20 An R^2 value of 0.9 or above is very good, a value above 0.8 is good, and a value of 0.6 or above may be satisfactory in some applications, although we must be aware of the fact that, in such cases, errors in prediction may be relatively high. When the R^2 value is 0.5 or below, the regression explains only 50% or less of the variation in the data; therefore, prediction may be poor [22].

Method to develop Multiple Quadratic Polynomial Regression Models and Quality Maps for Tensile Mechanical Properties and Quality Indices of Cast Aluminum Alloy according to Artificial Aging Heat Treatment Condition

In this subsection, a method to develop multiple quadratic polynomial regression models and quality maps for the tensile mechanical properties and quality indices of the cast aluminum alloys according to artificial aging heat treatment condition is proposed.

The main steps are as follows:

Step 1: Develop the multiple quadratic regression model for the tensile mechanical property (*YS*, *UTS*, E_f, *W*) and the quality index (Q, Q_R, Q_C, Q_0) according to the artificial aging heat treatment conditions (aging temperature T, aging time t) as the following form:

$$y = \beta_0 + \beta_1 \cdot T + \beta_2 \cdot t + \beta_3 \cdot T^2 + \beta_4 \cdot T \cdot t + \beta_5 \cdot t^2 \qquad (17)$$

This model is developed with data which consist of different artificial aging heat treatment conditions (aging temperature *T*, aging time *t*), tensile mechanical properties (*YS*, *UTS*, E_f, *W*) and quality indices (Q, Q_R, Q_C, Q_0) of the investigated cast aluminum alloys.

Step 2: Calculate and draw the lines of constant mechanical property (iso-mechanical property lines) or the lines of constant quality index (iso-quality index lines) using the corresponding regression model in 2D plane.

The iso-property lines or iso-quality index lines are calculated from the following equation:

$$\beta_0 + \beta_1 \cdot T + \beta_2 \cdot t + \beta_3 \cdot T^2 + \beta_4 \cdot T \cdot t + \beta_5 \cdot t^2 = C\ , \qquad (18)$$

where C is a fixed constant.

Step 3: Plot the pairs of aging temperature and time values of the investigated alloys in the above graph.

This graph is called quality map for the tensile mechanical property or quality index of the cast aluminum alloys. This map intuitively shows the quality levels of the cast aluminum alloys according to different heat treatment conditions (aging temperature and time).

Results and Discussion

Cast aluminium alloy A357 is artificially aged under different artificial aging heat treatment conditions. The evaluated tensile mechanical properties and quality indices of A357 with 27 different artificial aging heat treatment conditions are presented in Table 1 [2,11].

In this section, the multiple quadratic polynomial regression models and quality maps for the tensile mechanical properties and quality indices of the cast aluminum alloy A357 according to artificial aging heat treatment condition are developed with the data from Table 1.

Table 1: Mean values of the tensile mechanical properties and quality indices of cast aluminum alloy A357 according to 27 different artificial aging heat treatment conditions [2,11].

	Aging temperature	Aging time	Yield strength	Tensile strength	Elongation at fracture	Strain energy density	Quality indices			
	T (°C)	*t* (h)	*YS* (MPa)	*UTS* (MPa)	E_f (%)	*W* (MJ/m^3)	*Q* (MPa)	Q_R (MPa)	Q_C (MPa)	Q_0 (MPa)
1	155	6	160	277	19.52	49.38	470.5	1136.0	356.0	653.8
2	155	12	225	313	15.43	44.85	491.2	996.5	411.3	673.5
3	155	16	227	315	15.39	45.36	493.1	996.5	413.9	680.6
4	155	20	232	317	13.58	40.31	486.9	911.0	414.2	635.1
5	155	24	255	327	12.69	38.49	492.5	889.5	430.3	639.9
6	155	30	265	333	11.79	37.43	493.7	854.5	437.4	639.3
7	155	36	281	338	10.60	34.66	491.8	811.0	444.2	627.6
8	155	48	311	344	7.47	25.71	475.0	684.5	445.7	568.1
9	175	1	154	276	21.12	53.60	474.7	1210.0	354.4	690.0
10	175	3	190	292	17.68	48.36	479.1	1074.0	380.5	673.6
11	175	6	259	323	12.75	40.28	488.8	896.5	427.9	661.8
12	175	9	287	338	10.52	35.45	491.3	813.0	445.9	641.5
13	175	12	296	340	9.73	33.07	488.2	782.5	448.0	626.7
14	175	20	304	344	7.42	25.81	474.5	675.0	443.3	562.1
15	175	36	295	333	7.53	25.45	464.5	671.5	430.2	549.5
16	175	48	311	344	6.37	22.45	464.6	629.5	438.5	535.5
17	205	1	267	325	11.27	36.10	482.8	830.5	428.6	628.0
18	205	2	293	338	8.63	29.17	478.4	724.5	440.2	584.7
19	205	3	301	336	8.33	28.39	474.1	717.5	439.3	584.9
20	205	4	286	325	9.24	30.27	469.8	748.0	427.7	588.7
21	205	5	281	327	9.19	29.96	471.5	740.5	427.8	580.6
22	205	6	291	321	8.13	26.68	457.5	697.5	420.1	557.8
23	205	10	292	325	8.41	27.80	463.7	712.5	425.8	570.0
24	205	12	304	334	8.60	29.43	474.1	734.0	439.6	598.3
25	205	16	300	332	9.24	30.99	476.8	762.0	439.2	609.9
26	205	24	257	285	5.05	14.58	390.5	509.5	354.7	402.8
27	205	36	216	250	7.48	18.42	381.1	590.0	322.6	400.2

Multiple Quadratic Polynomial Regression Model and Quality Map for Tensile Mechanical Properties according to Artificial Aging Heat Treatment Condition

The multiple quadratic polynomial regression model and quality map for *YS* of A357 are shown in Eq. 18 and Figure 1.

$$YS = -2538.186432 + 26.970192 \cdot T + 32.097822 \cdot t - 0.064578 \cdot T^2 - 0.143513 \cdot T \cdot t - 0.111892 \cdot t^2 \quad (19)$$

The MAE, MRE and R^2 of this model are as follows:
MAE=15.976MPa, (20)
MRE=6.819%,
R^2=0.786.

By using Eq. 18 and Figure 1, it is possible to quantitatively and intuitively evaluate the *YS* value of the cast aluminum alloy A357 according to given artificial aging heat treatment condition and select or determine the rational artificial aging heat treatment conditions that satisfy the design prerequisites for *YS*. In the following Figures, the number of alloys refer to Table 1.

The multiple quadratic polynomial regression model and quality map for *UTS* of A357 are shown in Eq. 19 and Figure 2.

$$UTS = -1387.530241 + 17.131269 \cdot T + 20.193788 \cdot t - 0.042752 \cdot T^2 - 0.096759 \cdot T \cdot t - 0.060772 \cdot t^2 \quad (21)$$

The MAE, MRE and R^2 of this model are as follows:
MAE=9.529MPa,
MRE=3.052 %, (22)
R^2=0.748.

By using Eq. 19 and Figure 2, it is possible to quantitatively and intuitively evaluate the *UTS* value of the cast aluminum alloy A357 according to given artificial aging heat treatment condition and select or determine the rational artificial aging heat treatment conditions that satisfy the design prerequisites for *UTS*.

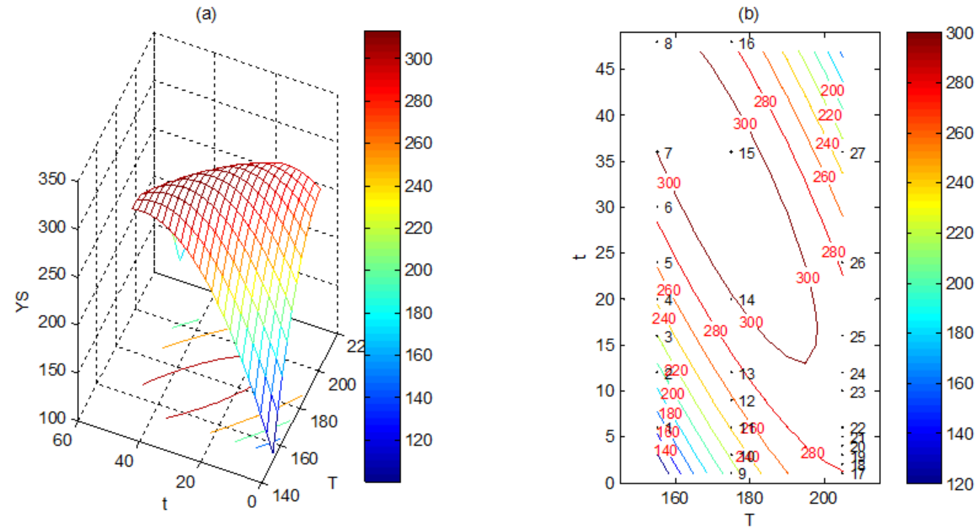

Figure 1: 3D surface graph of the regression model (a) and quality map (b) for *YS* of cast aluminum alloy A357 according to artificial aging heat treatment condition.

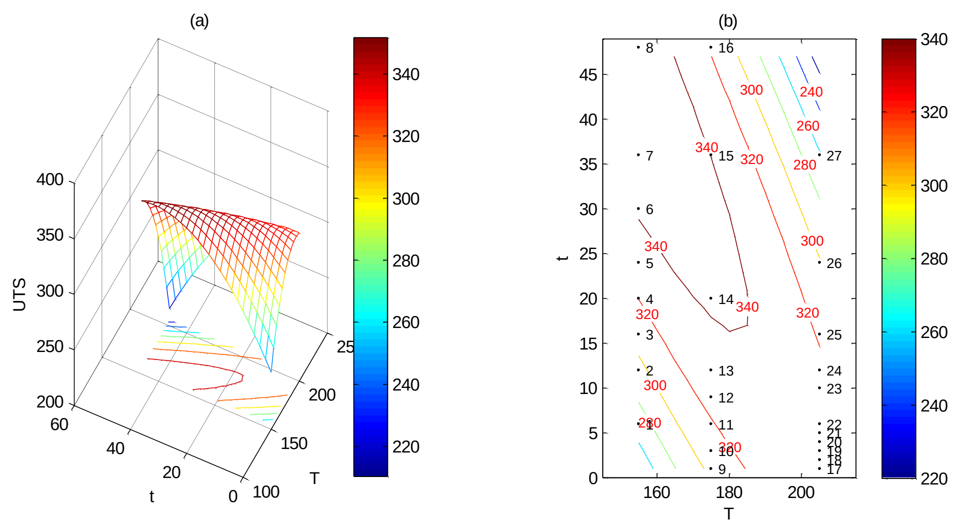

Figure 2: 3D surface graph of the regression model (a) and quality map (b) for *UTS* of cast aluminum alloy A357 according to artificial aging heat treatment condition.

The multiple quadratic polynomial regression model and quality map for E_f of A357 are shown in Eq. 20 and Figure 3.

$$E_f = 163.972973 - 1.373957 \cdot T - 1.703747 \cdot t + 0.003059 \cdot T^2 + 0.006477 \cdot T \cdot t + 0.007572 \cdot t^2 \quad (23)$$

The MAE, MRE and R^2 of this model are as follows:
MAE=1.224%,
MRE=11.795%, (24)
R^2=0.853.

By using Eq. 20 and Figure 3, it is possible to quantitatively and intuitively evaluate the E_f value of the cast aluminum alloy A357 according to given artificial aging heat treatment condition and select or determine the rational artificial aging heat treatment conditions that satisfy the design prerequisites for E_f.

The multiple quadratic polynomial regression model and quality map for *W* of A357 are shown in Eq. 21 and Figure 4.

$$W = 248.534506 - 1.758050 \cdot T - 2.157846 \cdot t + 0.003477 \cdot T^2 + 0.006329 \cdot T \cdot t + 0.011086 \cdot t^2 \quad (25)$$

The MAE, MRE and R^2 of this model are as follows:
MAE=2.793MJ/m^3, (26)
MRE=9.070%,
R^2=0.867.

By using Eq. 21 and Figure 4, it is possible to quantitatively and intuitively evaluate the *W* value of the cast aluminum alloy A357 according to given artificial aging heat treatment condition and select or determine the rational artificial aging heat treatment conditions that satisfy the design prerequisites for *W*.

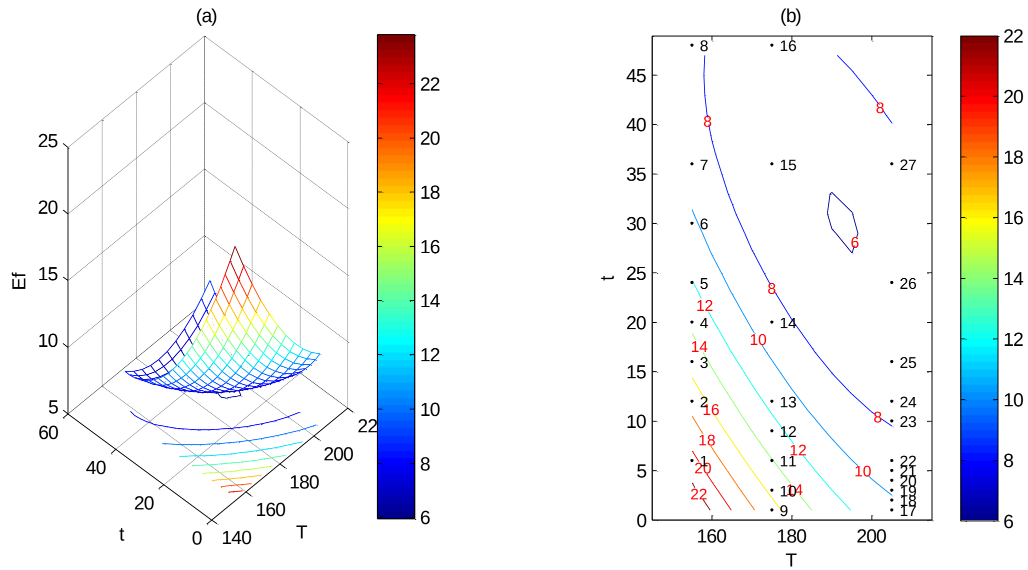

Figure 3: 3D surface graph of the regression model (a) and quality map (b) for E_f of cast aluminum alloy A357 according to artificial aging heat treatment condition.

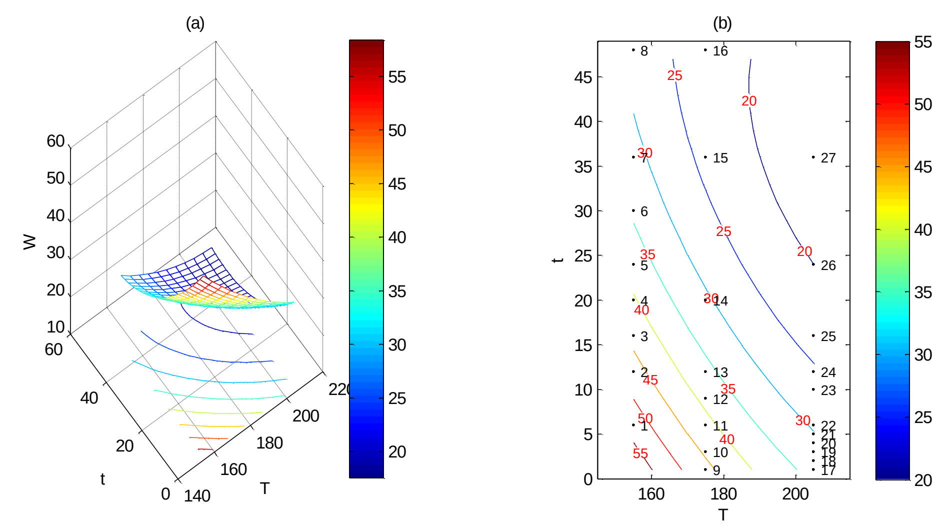

Figure 4: 3D surface graph of the regression model (a) and quality map (b) for *W* of cast aluminum alloy A357 according to artificial aging heat treatment condition.

Multiple Quadratic Polynomial Regression Model and Quality Map for Quality Indices according to Artificial Aging Heat Treatment Condition

The multiple quadratic polynomial regression model and quality map for *Q* of A357 are shown in Eq. 22 and Figure 5.

$$Q = -438.958673 + 9.912151 \cdot T + 13.043748 \cdot t - 0.026478 \cdot T^2 - 0.071426 \cdot T \cdot t - 0.029495 \cdot t^2 \quad (27)$$

The MAE, MRE and R^2 of this model are as follows:
MAE=7.598MPa, (28)
MRE=1.663%,
R^2=0.806.

By using Eq. 22 and Figure 5, it is possible to quantitatively and intuitively evaluate the *Q* value of A357 according to given artificial aging heat treatment condition and select or determine the rational artificial aging heat treatment conditions that satisfy the design prerequisites for *Q*.

The multiple quadratic polynomial regression model and quality map for Q_R of A357 are shown in Eq. 23 and Figure 6.

$$Q_R = 5660.462202 - 41.727683 \cdot T - 53.089517 \cdot t + 0.088387 \cdot T^2 + 0.180326 \cdot T \cdot t + 0.266725 \cdot t^2 \quad (29)$$

The MAE, MRE and R^2 of this model are as follows:
MAE=49.829MPa,
MRE=6.222%, (30)
R^2=0.856.

By using Eq. 23 and Figure 6, it is possible to quantitatively and intuitively evaluate the Q_R value of the cast aluminum alloy A357 according to given artificial aging heat treatment condition and select or determine the rational artificial aging heat treatment conditions that satisfy the design prerequisites for Q_R.

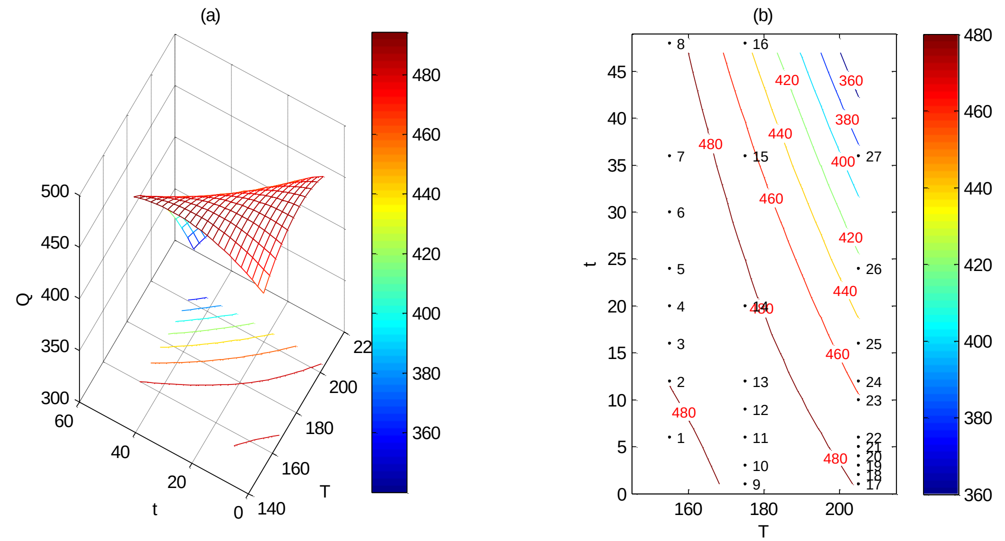

Figure 5: 3D surface graph of the regression model (a) and quality map (b) for *Q* of cast aluminum alloy A357 according to artificial aging heat treatment condition.

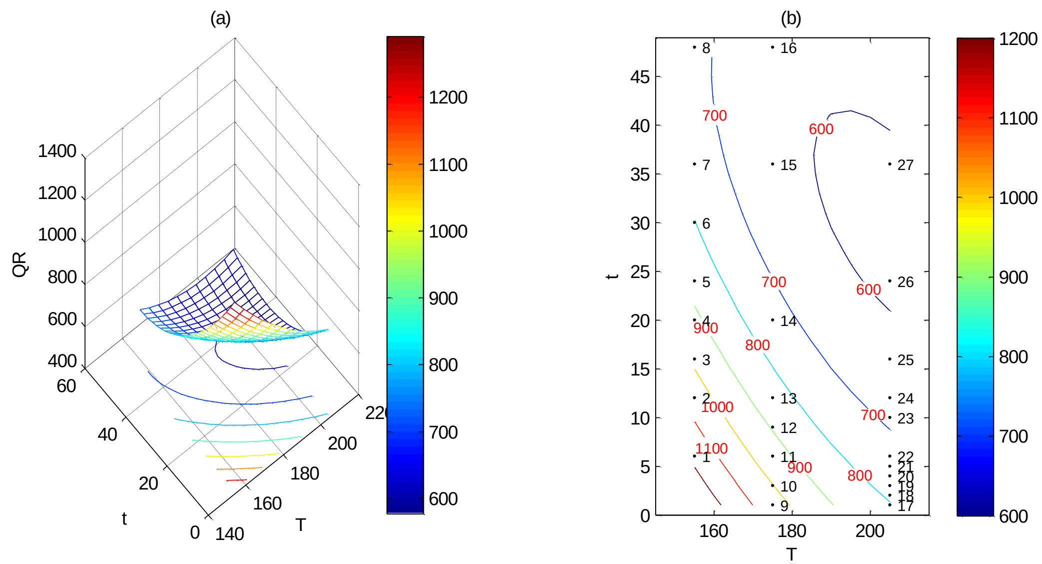

Figure 6: 3D surface graph of the regression model (a) and quality map (b) for Q_R of cast aluminum alloy A357 according to artificial aging heat treatment condition.

The multiple quadratic polynomial regression model and quality map for Q_C of A357 are shown in Eq. 24 and Figure 7.

$$Q_C = -1846.121845 + 22.618754 \cdot T + 28.431034 \cdot t - 0.056177 \cdot T^2 - 0.136343 \cdot T \cdot t - 0.089533 \cdot t^2 \tag{31}$$

The MAE, MRE and R^2 of this model are as follows:
MAE=13.456MPa,
MRE=3.313%, (32)
R^2=0.729.

By using Eq. 24 and Figure 7, it is possible to quantitatively and intuitively evaluate the Q_C value of the cast aluminum alloy A357 according to given artificial aging heat treatment condition and select or determine the rational artificial aging heat treatment conditions that satisfy the design prerequisites for Q_C.

The multiple quadratic polynomial regression model and quality map for Q_0 of A357 are shown in Eq. 25 and Figure 8.

$$Q_0 = -52.841375 + 9.389694 \cdot T + 10.519360 \cdot t - 0.029807 \cdot T^2 - 0.080221 \cdot T \cdot t - 0.001030 \cdot t^2 \tag{33}$$

The MAE, MRE and R^2 of this model are as follows:
MAE=20.395MPa,
MRE=3.626%, (34)
R^2=0.828.

By using Eq. 25 and Figure 8, it is possible to quantitatively and intuitively evaluate the Q_0 value of the cast aluminum alloy A357 according to given artificial aging heat treatment condition and select or determine the rational artificial aging heat treatment conditions that satisfy the design prerequisites for Q_0.

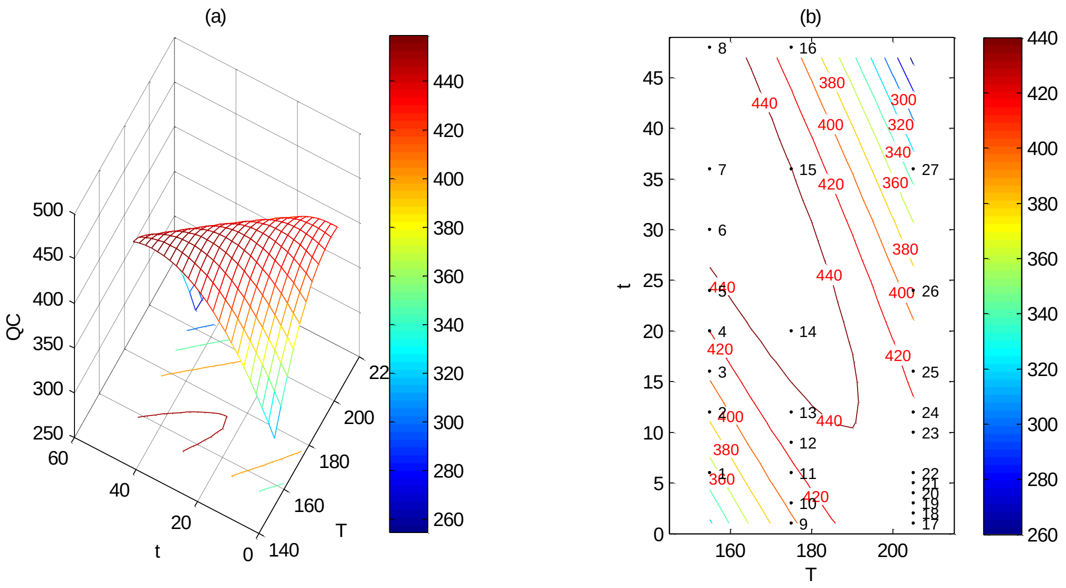

Figure 7: 3D surface graph of the regression model (a) and quality map (b) for Q_C of cast aluminum alloy A357 according to artificial aging heat treatment condition.

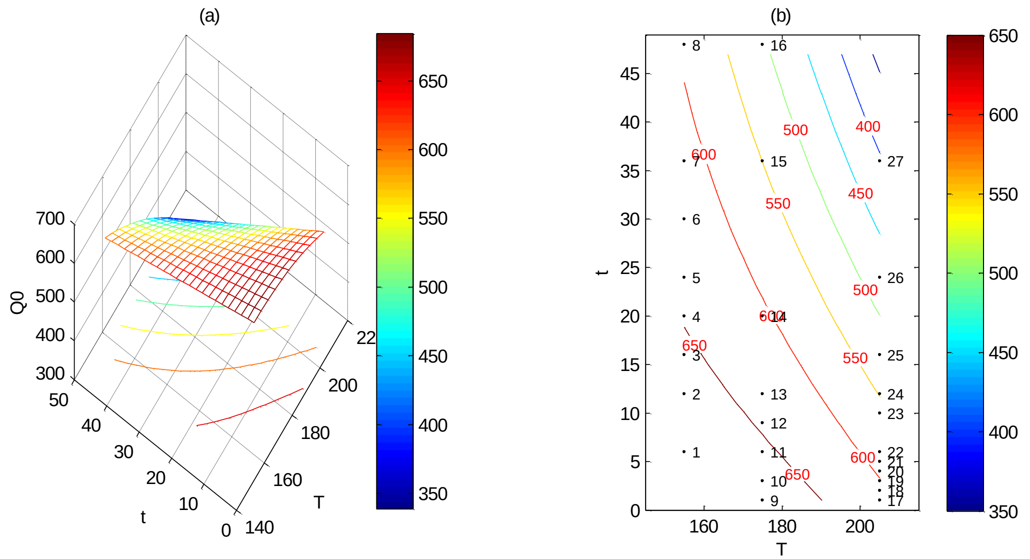

Figure 8: 3D surface graph of the regression model (a) and quality map (b) for Q_0 of cast aluminum alloy A357 according to artificial aging heat treatment condition.

In overall, the multiple quadratic polynomial regression models for tensile mechanical properties and quality indices according to artificial aging heat treatment condition are as follows:

$$YS = -2538.186432 + 26.970192 \cdot T + 32.097822 \cdot t - 0.064578 \cdot T^2 - 0.143513 \cdot T \cdot t - 0.111892 \cdot t^2$$

$$UTS = -1387.530241 + 17.131269 \cdot T + 20.193788 \cdot t - 0.042752 \cdot T^2 - 0.096759 \cdot T \cdot t - 0.060772 \cdot t^2$$

$$E_f = 163.972973 - 1.373957 \cdot T - 1.703747 \cdot t + 0.003059 \cdot T^2 + 0.006477 \cdot T \cdot t + 0.007572 \cdot t^2$$

$$W = 248.534506 - 1.758050 \cdot T - 2.157846 \cdot t + 0.003477 \cdot T^2 + 0.006329 \cdot T \cdot t + 0.011086 \cdot t^2$$

$$Q = -438.958673 + 9.912151 \cdot T + 13.043748 \cdot t - 0.026478 \cdot T^2 - 0.071426 \cdot T \cdot t - 0.029495 \cdot t^2$$

$$Q_R = 5660.462202 - 41.727683 \cdot T - 53.089517 \cdot t + 0.088387 \cdot T^2 + 0.180326 \cdot T \cdot t + 0.266725 \cdot t^2$$

$$Q_C = -1846.121845 + 22.618754 \cdot T + 28.431034 \cdot t - 0.056177 \cdot T^2 - 0.136343 \cdot T \cdot t - 0.089533 \cdot t^2$$

$$Q_0 = -52.841375 + 9.389694 \cdot T + 10.519360 \cdot t - 0.029807 \cdot T^2 - 0.080221 \cdot T \cdot t - 0.001030 \cdot t^2 \quad (35)$$

The corresponding quality maps are shown in Figures 1-8.

The MREs of the above regression models are less or slight larger than 10%, hence the regression models and quality maps are excellent accurate. The R^2 values of the above regression models are lager than 0.8 or 0.7, hence the regression models and quality maps are very good or may be satisfactory.

Comparison of the developed Models with Multiple Linear Regression Models and External Validation Results

Tables 2 shows the performance comparison result between the multiple quadratic and linear regression models for tensile mechanical properties and quality indices of A357.

As can be seen in Table 2, the performance of the multiple quadratic regression models are much superior than the multiple linear regression model. Therefore, it is reasonable to use the multiple quadratic polynomial regression models than multiple linear regression models for the tensile mechanical properties and quality indices of the cast aluminum alloy A357 according to artificial aging heat treatment condition.

Table 2: Comparison result of the multiple quadratic and linear regression models for tensile mechanical properties and quality indices of A357.

	Multiple quadratic regression models			Multiple linear regression models		
	MAE	MRE (%)	R^2	MAE	MRE (%)	R^2
YS (MPa)	15.976	6.819	0.786.	26.423	11.522	0.316
UTS (MPa)	9.529	3.052	0.748.	17.461	5.803	0.020
E_f (%)	1.224	11.795	0.853	1.515	14.722	0.719
W (MJ/m^3)	2.793	9.070	0.867	2.934	9.988	0.831
Q (MPa)	7.598	1.663	0.806	13.854	3.095	0.475
Q_R (MPa)	49.829	6.222	0.856	56.861	7.189	0.779
Q_C (MPa)	13.456	3.313	0.729	25.130	6.451	0.007
Q_0 (MPa)	20.395	3.626	0.828	24.213	4.499	0.752

For external validation of the multiple quadratic polynomial regression models for tensile mechanical properties of A357, the CV method is used. Tables 3 shows the CV results of the regression models for tensile mechanical properties and quality indices of A357 using CV method.

Table 3: CV result of the multiple quadratic regression models for tensile mechanical properties and quality indices of A357.

	MAE	MRE (%)	R^2
YS (MPa)	35.042	13.739	0.702
UTS (MPa)	16.788	5.414	0.726
E_f (%)	3.466	18.260	0.999
W (MJ/m^3)	8.028	18.654	0.995
Q (MPa)	11.250	4.865	0.993
Q_R (MPa)	89.245	14.335	0.898
Q_C (MPa)	21.577	7.435	0.976
Q_0 (MPa)	21.955	6.361	0.982

As can be seen in Table 3, the MREs are less than 20%, hence the performance of the multiple quadratic regression models are excellent and good accurate. The R^2 values are lager than 0.9 except *YS* and *UTS*, hence the regression models are very good. The R^2 values for *YS* and *UTS* are larger than 0.7, hence the regression models are satisfactory [20,22].

Conclusions

In this paper, the multiple quadratic polynomial regression models and quality maps for tensile mechanical properties and quality indices of the cast aluminum alloy A357 according to artificial aging heat treatment condition have been developed.
The developed multiple quadratic polynomial regression models and quality maps for tensile mechanical properties and quality indices of the cast aluminum alloy A357 evaluate the tensile mechanical properties and quality indices of the cast aluminum alloys according to artificial aging heat treatment condition with comparatively good accuracy. The multiple quadratic regression models may be widely used in practice because they have better performance than the multiple linear regression models and they are simpler than the artificial neural network models. By using the regression models and quality maps, materials designers and engineers can grasp the influences of the artificial aging heat treatment conditions to the tensile mechanical properties and quality levels, and decide the optimal or reasonable artificial aging heat treatment conditions in practice.

The proposed methodology may be applied to the other aluminum alloys.

The limitation of this work is that the work didn’t consider the problems to improve the quality of the prediction models by developing the optimal regression models for tensile mechanical properties and quality indices according to artificial aging heat treatment condition using artificial intelligent-based methodologies such as artificial neural network, metaheuristic-based machine learning and soft computing methods owing to limited space. There is something yet to study. Future work needs to study about the problems.

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