Book Chapter

Mechanical Characteristic Analysis of Buried Drainage Pipes after Polymer Grouting Trenchless Rehabilitation

Ren Wang$^{1,2,3,*}$, Jian-guo Xu$^{1,2,3}$ and Zhi-hao Chen$^{1,2,3}$

$^1$School of Water Conservancy Sci. & Eng. Institute of Underground Engineering, Zhengzhou University, China
$^2$National Local Joint Engineering Laboratory of Major Infrastructure Testing and Rehabilitation Technology, China
$^3$Collaborative Innovation Center of Water Conservancy and Transportation Infrastructure Safety, China

*Corresponding Author: Ren Wang, School of Water Conservancy Sci. & Eng. Institute of Underground Engineering, Zhengzhou University, Zhengzhou 450001, China

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**Abstract**

The application of Polymer grouting in underground pipeline rehabilitation is increasing gradually. The leakage and subsidence of buried pipelines could be repaired by polymer grouting technology. In order to analysis the calculation theory of the pipeline repairing process, the Winkler model and the Vlazov model of the pipe-soil-polymer interaction based on the elastoplastic theory is established, the calculation formulas of the pipe-soil interaction under polymer grouting is derived and the MATLAB calculation program based on the transfer matrix method is compiled. Then the calculated values are compared with the pipeline experimental values, and the influence of different factors on the internal force and deformation of the polymer-repaired pipeline under different work conditions is discussed. The results show that the values and trends of the pipe deformation and circumferential bending moment calculated by the models are consistent with the experimental results, and the results obtained by the Vlazov model are closer to the experimental values. In addition, the void at the bottom of the
pipeline has a large impact on the mechanical properties of the pipeline. However, polymer grouting can repair disengaged pipelines effectively and restore their mechanical properties. The proposed methods and calculation results are valuable for pipeline polymer repairing analysis and pipeline void repairing design.

**Introduction**

The urban drainage network used for discharging urban foul water and rainwater is very important for modern cities. However, most of the municipal drainage pipelines in China were established before the 1980s. As the service life is approaching, the pipelines are seriously aging, and they often have accidents such as disengagement, subsidence, corrosion and leakage. These pipelines cannot work properly, which leads to undesirable consequences, such as ground collapse and groundwater contamination [1]. The traditional way of repairing drainage pipelines is mainly overall excavation and then performing partial repairing or replacing the pipes. This method is expensive and has a serious impact on urban roads and surrounding residents. Polymer grouting technology is an effective trenchless rehabilitation technology for pipelines. It uses a self-expanding anhydrous reactive polymer material that expands and solidifies rapidly after reaction. The polymer grouting material is injected into the leaking area of the pipeline to fill the void outside the pipe and lift the settlement pipe (see Figures 1 and 2). At present, the technology has successfully reinforced and repaired various underground pipelines [2, 3].
For the mechanical properties of polymer-repaired pipelines, researchers often use finite element analysis software for simulation or experimental exploration. But the research on the pipe calculation theory and methods after pipeline repair is not enough. For example, Xu [4] used ABAQUS to simulate and calculate the stress of polymer-repaired pipelines. The results showed that the maximum tensile and compression stress of the pipes and the maximum vertical and horizontal displacements of the pipes decreased significantly after polymer grouting. Li [5] conducted a finite element analysis on the mechanical properties of repaired pipelines under vehicle loads, and the results showed that the stress and deformation of the disengaged pipes repaired by polymer restored to normal levels. Wang [6] conducted the field full-scale test was carried out for corrosion pipes with three different pipe bedding states under different vehicle axle loads,
then according to the field test, the refined three-dimensional finite element models of corroded drainage pipes with dense, void and polymer repaired pipe beddings were established, respectively, and the reliability of the test results was verified. Zhang [7] used finite element analysis software to analyse the effect of expansion and diffusion of polymer on pipe stress and strain, and the effect of polymer grouting repair on pipe stress and displacement. By comparing the simulation values with the experimental values, that can be matched the trends to prove the correctness and rationality of the numerical simulation values. Wang [8] conducted a full-scale experiment on polymer-repaired pipelines under impact loads and vehicle loads. The results showed that polymer grouting can effectively repair the void pipeline and even restore its mechanical properties to normal levels.

The buried pipes will deform because of overlaying soil and ground loads. Then, the left and right sidewalls of the pipe will squeeze the soil, which will cause the elastic resistance of soil to the pipeline and restrict pipe deformation and compensate for pipe rigidity. Therefore, when we study the calculation and analysis methods of buried pipes, the medium around the pipeline must be taken as a part of the pipe, and the pipe-soil interaction must be considered. Many research results have been done in this area. For example, Kjartanson [9] took the pipe as a series of beam elements, and took the backfill soil around the pipe as a nonlinear material. Base on this, a finite element analysis method for the pipe-soil interaction of buried pipes was proposed. Lee [10] used finite element models to reflect the soil characteristics, backfill and in-situ conditions, and selected soil models to measure the deflection of buried prestressed concrete cylinder pipes. Base on this and considering the pipe-soil interaction, a horizontal deflection equation of the underground pipeline was proposed. Jung [11] used the Mohr-Coulomb (MC) yield surface for peak strength to constrain soil deformation under strain and maximum uplift conditions, and the finite element model of pipe-soil interaction in granular soil is established. Burkov [12] presents the model and analysis of the stress-strain state of the soil-pipe interaction system, then analyzes the stress-strain state of the pipeline using the ANSYS.
The calculation results show that the stress-strain state of the pipeline depends on the depth of its location. Ke [13] used the Pasternak model to simulate the pipe-soil interaction and calculated the vertical displacement of the soil which caused by building a shield tunnel under the pipeline. The results showed the vertical displacement of the pipeline by 15.3% due to the shear force in the soil. Based on the Winkler foundation beam theory, Wang and Yao [14] gave the pipe-soil interaction model of buried steel pipes and the calculation formulas of the deflection and internal force of buried steel pipes. According to the measured results of buried pipeline stress characteristics, Liu and Yang [15] established a pipe-soil interaction analysis combination model and presented a method of determining the model parameters. The literature mostly focuses on the pipe-soil interaction, but for the polymer-repaired pipeline, the pipe-soil-polymer interaction should be considered. Therefore, a Winkler model and a Vlazov model considering the pipe-soil-polymer interaction are proposed based on the elastoplastic theory. Using the transfer matrix method, the MATLAB calculation program is compiled. By comparing the stress and deformation characteristics before and after polymer grouting repairing for underground pipeline to the experimental values, the accuracy and rationality of the Winkler model and Vlazov model are discussed. Finally, the Vlazov model is used to calculate the stress and deformation characteristics of the pipeline after polymer grouting repairing.

**Model of Pipe-Soil-Polymer Interaction Analysis**

**Winkler Model of Pipe-Soil-Polymer Interaction**

Winker proposes the idealized model of soil media. It assumes that the displacement at any point on the soil surface medium is proportional to the stress acting on that point, and is independent of the stress acting on other points [16]. The equation is as follows:

\[ q(x, y) = k \omega(x, y) \]  

(1)
where, \( q(x, y) \) is the soil surface pressure, \( \omega(x, y) \) is the soil surface settlement and \( k \) is the subgrade reaction coefficient.

Equation 1 is the response function of the Winker model. The model idealizes the soil medium as a series of independent spring element with a spring constant of \( k \). The soil and polymer are simplified into elastic materials to get the Winkler model of the pipe-soil-polymer interaction, which is shown in Figure 3.

![Figure 3: The Winkler model of pipe-soil-polymer interaction.](image)

Regarding the subgrade reaction coefficient \( k_1 \), many different calculation methods have been put forward, such as determining it through basic bearing plate experiments or determining it based on the experimental results of the California bearing ratio. By solving the uniform radial displacement on the circular boundary in an infinite elastic medium, the subgrade reaction coefficient \( k_2 \) can be obtained.

For an axisymmetric plane strain problem, the stress function is

\[
\Phi = A \ln r + Br^2 \ln r + Cr^2 + D [17],
\]

when ignoring body force, the equation is as follows:
In Equation 2, \( \Phi \) is the stress coefficient, \( A, B, C \) and \( D \) are the coefficients, \( r \) is the radius. \( \sigma_r \) and \( \sigma_\theta \) are the radial stress and tangential stress, respectively.

By the displacement single value property, the coefficient \( B=0 \), and at infinity from the origin, the stresses \( \sigma_r = \sigma_\theta \longrightarrow 0 \) from the Saint Venant's Principle, thus \( C=0 \).

Substitute \( B=0 \) and \( C=0 \) into Equation 2 to simplify and the equation is as follows:

\[
\begin{align*}
\sigma_r &= \frac{1}{r} \frac{\partial \Phi}{\partial r} = \frac{A}{r^2} + B(1 + 2 \ln r) + 2C \\
\sigma_\theta &= \frac{\partial^2 \Phi}{\partial r^2} = -\frac{A}{r^2} + B(3 + 2 \ln r) + 2C \\
\tau_{r\theta} &= 0 
\end{align*}
\] (2)

Substitute Equation 3 into Hooke's plane strain law, the equation can be described as

\[
\begin{align*}
\varepsilon_{r1} &= \frac{1}{E_g} (\sigma_r - \nu_g \sigma_\theta) = \frac{A(1 + \nu_g)}{E_g r^2} \\
\varepsilon_{r2} &= \frac{1}{E_s} (\sigma_r - \nu_s \sigma_\theta) = \frac{A(1 + \nu_s)}{E_s r^2} 
\end{align*}
\] (4)

where \( \varepsilon_{r1} \) and \( \varepsilon_{r2} \) are the strain of the polymer layer and soil layer, respectively. \( E_g \) and \( \nu_g \) are the elastic modulus and Poisson ratio of polymer, respectively. \( E_s \) and \( \nu_s \) are the elastic modulus and Poisson ratio of soil, respectively. The uniform
radial displacement on the circular boundary is calculated by the following equation:

\[
  u = \int_{R}^{R+D} \varepsilon_{r1} dr + \int_{R}^{\infty} \varepsilon_{r2} dr
\]  

(5)

where \( D \) is the polymer layer thickness.

Substitute Equation 4 into the Equation 5 to get the radial displacement:

\[
  u = \frac{A}{R+D} \left( \frac{1+v_s}{E_s} - \frac{1+v_g}{E_g} \right) + \frac{A(1+v_g)}{E_g R}
\]  

(6)

With the elastic parameters \( k \) defined in the Winkler model and Equations. 3 and 6, the subgrade reaction coefficient of the polymer layer is:

\[
  k_2 = \frac{k_s}{u} = \frac{1}{R^2 \left( \frac{1+v_s}{E_s} - \frac{1+v_g}{E_g} \right) + \frac{R(1+v_g)}{E_g}}
\]  

(7)

**Vlazov Model of Pipe-Soil-Polymer Interaction**

Because the Winkler model has inherent defects in describing the continuity of natural soil, it is suitable for the case where the foundation soil (compression layer) is thin. The more the compression layer is thick, the greater error will become. To make up for the defects of the Winkler model, researchers get a new two-parameter model by introducing constraints in the elastic continuous medium model or simplifying certain assumptions of displacement distribution and stress [16]. The Vlazov model is one of them and it is based on variational calculus. The model formula by introducing some displacement constraints to simplify the basic equations of the isotropy linear
elasticiy continuous medium model and the equation is as follows:

$$q(x, y) = K\omega(x, y) - T\nabla^2 \omega(x, y)$$

(8)

where $q(x, y)$ is the soil surface pressure, $\omega(x, y)$ is the soil surface settlement, $\nabla^2$ is the Laplace operator, $K$ is the measure of soil deformation under compression stress, $T$ is the measure of soil unit transitivity.

The model is also applicable to non-uniform elastic layers. Simplify both the soil and polymer into elastic materials and then the Vlazov model for pipe-soil-polymer interaction analysis can be obtained as shown in Figure 4.

![Figure 4: The Vlazov model of pipe-soil-polymer interaction.](image)

The model parameters can be obtained by solving the subgrade control equations of the two-parameter model. Under the plane strain conditions, in the $x-z$ plane, the displacement $M$ of any point in the soil layer is:

$$\bar{u}(x, z) = 0, \bar{\omega}(x, z) = \omega(x)\varphi(z), [\varphi(0) = 1, \varphi(H) = 0]$$

(9)

where $\varphi(z)$ is a change function to describe the displacement $\omega(x, y)$ in the direction $z$. 
Based on the relationship between the stress and strain under the plane strain conditions, the equation is as follows:

\[
\begin{align*}
\sigma_{z1} &= \frac{E_1}{1-\nu_1^2} \omega(x) \varphi'(z) \\
\tau_{xz1} &= \frac{E_1}{2(1+\nu_1)} \omega'(x) \varphi(z) \\
\sigma_{zz} &= \frac{E_2}{1-\nu_2^2} \omega(x) \varphi'(z) \\
\tau_{xz2} &= \frac{E_2}{2(1+\nu_2)} \omega'(x) \varphi(z)
\end{align*}
\]  

(10)

where \( E_1 = E_g / (1-\nu_g^2), \nu_1 = \nu_g / (1-\nu_g) \), \( E_2 = E_s / (1-\nu_s^2) \), \( \nu_2 = \nu_s / (1-\nu_s) \), \( E_g \) and \( \nu_g \) are the elastic modulus and the Poisson ratio of polymer, respectively, \( E_s \) and \( \nu_s \) are the elastic modulus and the Poisson ratio of soil, respectively.

According to Lagrange virtual work principle, the virtual displacement is \( \delta \bar{u} = 0 \), \( \delta \omega = \varphi(z) \delta \omega \). The virtual work done by the external force is:

\[
U_e = bq(x) \varphi(0) \delta \omega dx + b \int_0^{H_1} \frac{\partial \tau_{xz1}}{\partial x} \varphi(z) \delta \omega dx dx + b \int_{H_1}^H \frac{\partial \tau_{xz2}}{\partial x} \varphi(z) \delta \omega dx dz
\]

(11)

The virtual work done by the internal force is:

\[
U_i = -b \int_0^{H_1} \sigma_{z1} \varphi'(z) \delta \omega dx dz - b \int_{H_1}^H \sigma_{zz} \varphi'(z) \delta \omega dx dz
\]

(12)

Substituting Equation 10 into \( U_e + U_i = 0 \), the equation can be described as:

\[
q(x) = K \omega(x) - 2T \frac{d^2 \omega(x)}{dx^2}
\]

(13)

where
For functions \( \varphi(z) \), Vlazov proposed linear and nonlinear expressions, they are: \( \varphi_1(z) = 1 - z/H \),

\[
\varphi_2(z) = \frac{sh[\gamma(H - z)/L]}{sh[\gamma h/L]}.
\]

where \( H \) is the thickness of the elastic layer used for calculation, \( L \) is the length of the buried pipeline used for calculation, \( \gamma \) is the constant used to reflect the foundation beam characteristics, which is usually 1.5. In this paper, the linear expression is used for the polymer layer, and the nonlinear expression is used for the soil layer. Substitute them into the calculation formulas of \( K \) and \( T \), the model parameters are written as follows:

\[
K_j = \frac{E_j}{H_j(1 - v_j)} \left( \frac{\gamma H}{2L} \left( \frac{sh(\gamma H / L)ch(\gamma H / L) + \gamma H / L}{sh^2(\gamma H / L)} \right) \right)
\]

\[
T_j = \frac{E_jH}{12(1 + v_j)} \left( \frac{3L}{2H} \left( \frac{sh(\gamma H / L)ch(\gamma H / L) - \gamma H / L}{sh^2(\gamma H / L)} \right) \right)
\]

Analysis of Pipeline Strain Problems based on the Transfer Matrix Method

Transfer Matrix Method

The transfer matrix method uses matrix multiplication to find the internal force and displacement of each section, based on the basic differential equation of the structure, performs mechanical
analysis of the structure [18]. Because of using simple matrix multiplication, it is suitable for calculations using computer programming. Compared with the finite element method, the transfer matrix method prevents the overall stiffness matrix from being too large, which makes it difficult to solve large linear systems [19]. At the same time, it does not need to select a suitable soil constitutive model and set up contact elements, which will make calculation simply. Therefore, this method has simple calculations. The calculation process can be expressed by the following formula:

\[ S_i = U_i U_{i-1} \cdots U_2 U_1 S_0 \]  

(14)

where \( S_0 \) is the initial situation vector, \( S_i \) is the end situation vector, \( U_1, U_2, \ldots, U_i, U_{i-1} \) is the transfer matrix of each part.

**Description of the Pipeline Calculation Model**

Due to the existence of the pipe joints, the pipeline structure is discontinuous. It is difficult to theoretically consider the interaction of each pipe section and joints. In view of the problems studied in this article, it is necessary and feasible to simplify the model by ignoring some secondary factors. At the non-joints, we simulate the pipeline as a horizontally homogeneous circular ring and a longitudinal continuous cylinder. Then we assume that the length of pipeline is long enough and the external load is evenly distributed along the axial direction of the pipe. Therefore, the calculation of pipeline stress and deformation can be regarded as a plane strain problem. At the joints, according to the longitudinal equivalent continuity model proposed by Shiba Yukio [20-21], the connection effect of joints and pipe section can be considered by equivalent stiffness, then calculated according to the above method. In the paper, to simplify the calculation, we only consider the pipe at the non-joints. The load acting on the pipeline is considered to be symmetrical, so only half of the pipeline is calculated and analyzed. As shown in Figure 5, a unit length pipeline is chosen. The wall thickness and radius of the pipeline are \( t \) and \( R \), respectively. Half of the pipeline is divided into \( n \) units. At the
same time, the pipe-soil deformation tendency and the soil compaction degree at different positions are different. So we divide the polymer and soil into three parts, namely the bottom support area, the chest cavity area and the pipe crown area.

![Figure 5: Calculation model of a buried pipe.](image)

We assume that all the divided elements are beam ones and are rigidly connected in sequence, and describe the interaction between each element and the polymer or soil through the pipe-soil-polymer interaction analysis model. Then we can get the transfer matrix of the beam element based on the basic differential equation of the force structure in Material Mechanics. After calculate the transfer matrix of each unit, according to the boundary conditions, the internal force and deformation of each pipe section base on the transfer matrix method can be obtained.

Before calculate the pipe stress and deformation, the pipe additional load. It has many parts should be known, such as vertical soil pressure, horizontal soil pressure, the ground load, internal and external hydrostatic pressure.

Soil pressure calculation equation is as follows:

\[
\sigma_v(i) = C_v (\gamma H_w + \gamma_d (H - H_w + R - Y(i)))
\]  
\[
\sigma_h(i) = k \sigma_v(i)
\]  

(15)
Where: $\sigma_v(i)$ and $\sigma_h(i)$ are vertical soil pressure and horizontal soil pressure, $H$ and $H_w$ are pipe depth and groundwater level depth, $\gamma$ and $\gamma_d$ are dry soil bulk density and saturated soil bulk density, $R$ is pipe radius, $Y(i)$ is $Y$ coordinate at node $i$, $k$ is lateral pressure coefficient of soil pressure. $C_v$ is soil pressure concentration factor, in this paper, which determined based on Liu's research results [22].

The ground load is the general term for all loads acting on the ground. It mainly includes traffic load and building load, the pipe pressure caused by the ground load can be calculated according to the Bousinesq [17] formula. Hydrostatic pressure calculation equation is as follows:

$$
\sigma_{wo} = \gamma_w (H - H_w + R - Y(i))
$$
$$
\sigma_{wi} = \gamma_w (R - Y(i))
$$

(16)

Where, $\sigma_{wo}$ and $\sigma_{wi}$ are external and internal hydrostatic pressure, $\gamma_w$ is water bulk density.

Considering all the above forces and through the transform matrix, the radial and tangential loads at node $i$ can be calculated. The force diagram of unit $i$ is shown in Fig. 6.

![Figure 6: Force diagram for the element.](image)
In Figure 6, $p_i$ and $q_i$ are the radial and tangential loads at node $i$, respectively. $p_{i+1}$ and $q_{i+1}$ are the radial and tangential loads at node $i+1$, respectively. $l$ is the length of unit $i$. $\theta$ is the angle between unit $i$ and $X$ axis in the overall coordinate system.

Under the action of additional force, assume that the tangential displacement at node $i$ of unit $i$ is $u_i$, the radial displacement is $v_i$, the rotation angle is $\omega_i$, the bending moment is $M_i$, the axial force is $N_i$, and the shear force is $T_i$. The interaction between unit $i$ and the polymer or soil is described by the pipe-soil-polymer interaction analysis Vlazov model. Then in the local coordinate, according to equilibrium conditions and physical equations, the formulas for calculation are as follows:

$$
\begin{align*}
\frac{dT}{dx} &= -K' y + 2T' \frac{d^2 y(x)}{dx^2} - p_i \left(1 - \frac{x}{l}\right) - p_{i+1} \frac{x}{l}
\frac{dN}{dx} &= q_i \left(1 - \frac{x}{l}\right) + q_{i+1} \frac{x}{l}
\frac{dM}{dx} &= T
\frac{du}{dx} &= -\frac{N}{ES}
\frac{d^2 v}{dx^2} &= \frac{M}{EI}
\frac{dv}{dx} &= \sigma
\end{align*}
$$

where, $K'$ and $T'$ are Vlazov model parameters, $E$ is pipe elastic modulus, $S$ is inertia moment of pipe section, $I$ is cross-sectional area of pipe element.

By solving the above Equation 17, and with the boundary conditions: $u = u_i$, $v = v_i$, $\omega = \omega_i$, $M = M_i$, $N = N_i$, $T = T_i$.
when \( x = 0 \), and \( u = u_{i+1} \), \( v = v_{i+1} \), \( \omega = \omega_{i+1} \), \( M = M_{i+1} \), \( N = N_{i+1} \), \( T = T_{i+1} \) when \( x = l \), the formula for calculation is as follows:

\[
\{S_{i+1}\} = [U_{i,i+1}]\{S_i\}
\]  \hspace{1cm} (18)

where \( \{S_i\} = \{u_i, v_i, \omega_i, M_i, N_i, T_i, 1\} \) is the matrix of the internal force and displacement at node \( i \), \([U_{i,i+1}]\) is the transfer matrix of \( 7 \times 7 \), \( \{S_{i+1}\} = \{u_{i+1}, v_{i+1}, \omega_{i+1}, M_{i+1}, N_{i+1}, T_{i+1}, 1\} \) is the matrix of the internal force and displacement at node \( i + 1 \).

The transfer matrix, internal force and displacement matrix are all belong to the local coordinate. When calculating the internal force and displacement of each element, the transfer matrix needs to be converted into global coordinates. Through the coordinate transform matrix \( R \), we can transform Equation 18 into global coordinates as shown in the following equation:

\[
\{\overrightarrow{S}_{i+1}\} = [\overrightarrow{U}_{i,i+1}]\{\overrightarrow{S}_i\}
\]  \hspace{1cm} (19)

where \( [\overrightarrow{U}_{i,i+1}] = R[U_{i,i+1}]R^{-1} \) is the transfer matrix of unit \( i \), \( \{\overrightarrow{S}_i\} \) and \( \{\overrightarrow{S}_{i+1}\} \) are the matrices of the internal forces and displacements at node \( i \) and node \( i + 1 \) in the global coordinate, respectively. \( R \) is the coordinate transform matrix of unit \( i \).

With the transfer matrix of each unit, the following equation by substituting it into Equation 14 can be written as follows:

\[
\{\overrightarrow{S}_n\} = [\overrightarrow{U}_{n,n+1}] [\overrightarrow{U}_{n-1,n}] \cdots [\overrightarrow{U}_{2,3}] [\overrightarrow{U}_{1,2}] \{\overrightarrow{S}_1\}
\]  \hspace{1cm} (20)

where \( \{\overrightarrow{S}_1\} \) and \( \{\overrightarrow{S}_n\} \) are the matrices of the internal forces and displacements at node 1 and node \( n \).
Since the load is symmetrical, according to the constraints of boundary conditions, $u_1 = \omega_1 = T_1 = 0$, $u_n = \omega_n = T_n = 0$ at node 1 and node $n$, $\nu_1$, $M_1$, $N_1$ and $\nu_n$, $M_n$, $N_n$ can be calculated with Equation 20.

Then taking node 1 as a known quantity and substituting it into Equation 21, the internal force and displacement of any node can be calculated and the Equation is as follows:

$$\{S_i\} = \{U_{i,i+1}\}[\{U_{i,i-1}\}] \cdots [\{U_{1,2}\}]\{S_1\}$$

(21)

After calculating the displacement of each node, the corresponding contact pressure of each element can be determined. Combining the soil elastoplastic deformation law and the pipe deformation, a part of the contact pressure is limited to meet the plasticity conditions of the soil and the iterative calculation is used to meet all the boundary conditions and deformation criteria.

**Program Implementation**

According to the above calculation process, MATLAB is used to compile the pipeline stress calculation program. The calculation flowchart is as follows:
Enter basic parameters: pipeline parameters, soil parameters, external loads, etc

Calculate the load acting on pipeline

Calculate the transfer matrix of each unit according to formula

Find internal force and displacement of the initial node according to boundary conditions

Find internal force and displacement of each node one by one according to treating initial point as a known point

- Whether the contact pressure of each element meets soil elastic-plastic deformation rule
  - Y: Output result: internal force and displacement of each node
  - N: Modify the contact pressure to meet soil plastic condition

Modify the contact pressure to meet soil plastic condition

Recalculate the load acting on pipeline

Figure 7: Flow chart of the pipeline stress calculation program.

Experimental Description and Data Processing

The experiments are carried out in a relatively open outdoor area. A relatively flat place is selected. The experimental pits, the length, width and depth of which are 8 m, 2.3 m and 2.3 m, respectively, are excavated using small equipment. Due to over-excavation and soil disturbance at the bottom of the pit, the undisturbed soil at the bottom is layered and compacted to a depth of 0.36 m below the bottom of the pit before laying the pipeline. Three complete tube sections and two half-tube sections are selected, which are connected through bell and spigot joints (rubber rings are used as seals to prevent leakage). The effective length is 8 m. After the pipe is in place, the trench is backfilled with several layers of loose soil, and the thickness of each layer is about 0.2 m. The soil is then compacted by a small rammer. Then perform a normal, empty, polymer repair
condition test on the same pipeline. At void area, when the pipeline has no leakage and the sinking pipe section is raised to the design elevation after polymer repair, we think buried drainage pipes can work properly.

The test pipe failure criteria based on GB/T 50332 (2002) [23], each pipe segment having a nominal inner diameter of 700 mm, wall thickness of 70 mm, and an effective length of 2000 mm. The strength of the concrete used for pipe making was C30, and the standard value of its compressive strength was 30 MPa. The internal and external strains of the pipe under load are measured by attaching strain gauges to the inner and outer walls of the pipe. During the tests, the mechanical responses of the pipe to the loads are small, far from reaching the ultimate strength of the material. Therefore, the pipeline can be considered to be still in its elastic phase during the tests. The performance parameters of the pipeline, soil and polymer involved in the experiments are shown in Table 1. The experimental pipeline cross-section and the field experimental diagram are shown in Figure 8.

**Table 1:** Performance parameters of the pipeline, soil and polymer used in the experiment.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density $\rho$ / Kg·m$^{-3}$</th>
<th>Elastic modulus $E$ / MPa</th>
<th>Poisson ratio</th>
<th>Cohesive force $C$ / kPa</th>
<th>Internal friction angle $\varphi$</th>
<th>Dilatancy angle $\Delta \varphi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe</td>
<td>2400</td>
<td>30000</td>
<td>0.3</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Soil</td>
<td>1980</td>
<td>60</td>
<td>0.3</td>
<td>40</td>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>Polymer</td>
<td>240</td>
<td>20</td>
<td>0.2</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
Figure 8: Sectional view of the experimental pipeline and the pictures of the field experiment: (a) plan view, (b) north-south section view (through the cross-section of the pipeline), (c) east-west section view (unit: meter), (d) and (e) are pictures of the field experiment.

The experimental data should be processed to better compare the experimental value with the calculation result of the model. The relationship between the pipe bending moment and the strain of the inner and outer pipe walls is described as follows [24]:

\[
M = EI \left( \frac{\varepsilon_{\text{in}} - \varepsilon_{\text{out}}}{h} \right)
\]

(22)

where \(M\) is the pipe bending moment, \(E\) is the pipe elastic modulus, \(I\) is the moment of inertia, \(\varepsilon_{\text{in}}\) and \(\varepsilon_{\text{out}}\) are the internal and external pipe strain, respectively and \(h\) is the pipe thickness.

In the pipeline design, the ability of a pipeline to withstand external loads is defined by the pipe stiffness, which is related to the geometry and material of pipes. At the same time, in ASTM D 2412-11[25], the pipe stiffness is also defined as: the ratio of the load that causes a certain percentage of pipe deformation (5%) to the pipe inner diameter deformation along the load direction. Therefore, the pipe radial deformation can be calculated by
$$\Delta y = \frac{F}{PS}$$  \hspace{1cm} (23)

where $\Delta y$ is the pipe radial deformation, $F$ is the pipe radial stress and $PS$ is the pipe stiffness.

**Comparing Model Calculation Results with Experimental Results**

According to the experiment, the static load acting pipe section is taken as the calculation pipeline, and different pipe-soil-polymer interaction models are used for simulation. The calculation results are shown in Figure 9 and Figure 10. Figure 9 presents the radial deformation and circumferential axial force calculated by the Winkler model under normal, disengaged and polymer-repaired conditions. Figure 10 presents the results calculated by the Vlazov model. It can be seen from Figure 9 and Figure 10 that the radial deformation of the pipe after polymer repair is slightly larger than the normal value, and the circumferential axial force is basically the same as the normal value. The maximum radial deformation difference under normal and polymer-repaired appears at bottom of the pipe and it is 0.4 mm in Figure 9a and 1 mm in Figure 10a. The deformation and mechanical properties of pipes after polymer grouting can be restored to normal levels. Under disengaged conditions, the radial deformation at the bottom and the circumferential axial force at the crown will increase significantly. Compared with the normal situation, the radial deformation at the bottom increases by 3.9 times (see Figure 9a) and 5.3 times (see Figure 10a), and the circumferential axial force at the crown increases by 1.6 times (see Figure 9b) and 2.0 times (see Figure 10b), respectively. It shows that the bottom void has a great influence on the mechanical properties of the pipe. At the same time, as shown in Figure 9 and Figure 10, the radial deformation of the entire pipe will increase in a disengaged state, because the lack of soil at the bottom can change the pipe constraints. Therefore, the pipe deformation will increase when a load is applied.
Figure 11 presents the comparison of the radial deformation and the circumferential bending moment calculated by different analysis models with the experimental values. Table 2 lists the parameters of the pipe-soil-polymer interaction analysis model under normal and polymer repair conditions. Table 3 lists the sample point data of the experiment. As shown in Table 2, the coefficient of the subgrade reaction after polymer repair calculated by the Winkler model is 66% of the normal value, and that calculated by the Vlazov model is 91% of the normal value. It shows that the results calculated by the Vlazov model are closer to normal. As shown in Figure 11, the radial deformation and the circumferential bending moment calculated by the models are basically consistent with the experimental values, and the maximum radial deformation difference is 2 mm compared with the experimental values. The small error indicates that the calculation method is reliable. The calculation result of the Vlazov model is obviously larger than that of the Winkler model. This is because the Vlazov model increases the soil lateral force transmission so that the load transferability of the soil medium is more reasonable than that of the Winkler model. But compared with the experimental value, the model calculation will have a certain error. This is because in the calculation, we assume that the soil is an elastic body, but the soil is not completely elastic. On the other hand, we assume that the polymer repair area is the same length as the pipeline in the longitudinal direction, but the length of the polymer repair area will be relatively short in the engineering practical projects.

As shown in Figure 11a, the radial deformation calculated by the Vlazov model is close to the experimental value. Except for the pipe crown, the calculation error is between 2% and 25%. At the 45° haunch, the model calculation result is closest to the experimental value. The calculation error of the Winkler model is relatively large, which is between 9% and 55%. As shown in Figure 11b, regarding the calculation of the circumferential bending moment, the errors between the calculation results of the Winkler model and experimental values are 35%, 21% and 12% at the bottom, spring line and crown, respectively. The errors between the calculation results of the Vlazov model and the experimental values are 18%, 16% and 11%, respectively.
Therefore, the circumferential bending moment calculated by the Vlazov model is closer to the experimental value. In summary, compared with the Winkler model, the Vlazov model is more consistent with the actual pipe stress.

![Graph A: Radial deformation](image)

![Graph B: Circumferential axial force](image)

**Figure 9:** Calculation results of the Winkler model: (a) Radial deformation,(b) Circumferential axial force.
Figure 10: Calculation results of the Vlazov model: (a) Radial deformation,(b) Circumferential axial force.
Table 2: Model parameters of the pipe-soil-polymer interaction analysis.

<table>
<thead>
<tr>
<th>Parameters of the pipe-soil-polymer interaction model</th>
<th>The bottom support area (polymer-repaired)</th>
<th>The bottom support area (normal)</th>
<th>polymer-repaired / normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winkle model K/N·m$^{-3}$</td>
<td>68945</td>
<td>105230</td>
<td>66%</td>
</tr>
<tr>
<td>Vlazov model K/N·m$^{-3}$</td>
<td>82842</td>
<td>91434</td>
<td>91%</td>
</tr>
<tr>
<td>Vlazov model T/N·m$^{-1}$</td>
<td>66</td>
<td>91</td>
<td>73%</td>
</tr>
</tbody>
</table>

Figure 11: Comparison of the model calculation results and the experimental values (after polymer-repaired): (a) Radial deformation, (b) Circumferential bending moment.
Table 3: The sample point data of the experiment

<table>
<thead>
<tr>
<th>Sample point (degree from the pipe bottom) /°</th>
<th>Internal pipe strain/ με</th>
<th>External pipe strain/ με</th>
<th>Pipe radial stress/ kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.868</td>
<td>1.503</td>
<td>63.201</td>
</tr>
<tr>
<td>45</td>
<td>5.174</td>
<td>3.842</td>
<td>76.324</td>
</tr>
<tr>
<td>90</td>
<td>5.203</td>
<td>0.701</td>
<td>50.504</td>
</tr>
<tr>
<td>135</td>
<td>-6.012</td>
<td>-4.507</td>
<td>-100.761</td>
</tr>
<tr>
<td>180</td>
<td>-6.678</td>
<td>-3.958</td>
<td>-135.723</td>
</tr>
</tbody>
</table>

Influence of Different Factors on the Internal Force and Deformation of Pipelines after Polymer Repair

The internal force and deformation of a buried pipeline are affected by many factors, such as the depth, diameter and material of the pipeline, the ground load and the nature of surrounding filler. The pipe-soil-polymer interaction simulated by the Vlazov model is more in line with the actual situation. So the Vlazov model is used to calculate the pipeline after repaired with polymer grouting under different working conditions to explore the influence of different factors on the internal force and deformation after repair.

Influence of Soil Cover Depth

Figure 12 and figure 13 show the radial deformation and the circumferential bending moment after polymer-repair when the soil cover depth is 1 m, 1.5 m and 2 m, respectively. At different cover depths, the radial deformation and bending moment change trends are consistent. As the cover depth increases, its value will increase significantly. This is because the overburden pressure of the pipe will increase as the cover depth increases, which will result in increased stress and deformation. In Figure 12, as different coordinate systems are used, although the bottom deformation is positive and the crown is negative, both are downward and the deformation at the crown is greater than that at the bottom. As shown in Figure 13, the maximum value of the circumferential bending moment appears at the spring line.
Influence of different Load

Figure 14 and figure 15 show the radial deformation and the circumferential bending moment after different loads are applied to the pipe crown. As the additional load increases, the circumferential bending moment and the radial deformation of the crown increase significantly. As shown in Figure 14, when the load is 0.1 MPa and 0.15 MPa, the radial deformation of the bottom is 1.1 times and 1.2 times that the load is 0.05 MPa, respectively. Similarly, the radial displacement of the crown is 1.3 times and 1.5 times that the load is 0.05 MPa. With the same load change, the deformation change rate of the crown is greater than that of the bottom. This is because the soil cover depth of the bottom is deeper than that of the crown. As the soil cover depth increases, compared with the earth pressure, the effect of the additional load becomes smaller and the pipe-soil interaction is strengthened, which limits the pipe deformation to a certain extent.

Figure 12: Radial deformation at different buried depths.
Figure 13: Circumferential bending moments at different buried depths.

Figure 14: Radial deformation with different loads.
Figure 15: Circumferential bending moments with different loads.

Figure 16 and figure 17 show the radial deformation and the circumferential bending moment when the soil elastic modulus is changed. As shown in Figure 16, below of spring line, the radial deformation of the polymer-repaired area increases with the increase of the backfill soil elastic modulus, while it shows the opposite trend in the non-void area. We can know that the pipe-soil interaction in the non-void area is strengthened when the soil elastic modulus increases. But in the polymer-repaired area, the composite soil layer is formed due to the addition of polymer. The interaction between the pipeline and the composite soil layer weakens as the soil elastic modulus increases. As shown in Figure 17, in the non-void area, the bending moment of the crown and the spring line lower have opposite change trend, because the change is the result of the combined effect of the two aspects [26]. Increasing the elastic modulus of the soil around the pipe will increase the elastic resistance of the soil to the pipeline, strengthen the pipe-soil interaction, and make the pipe internal force distribution more uniform, thereby reducing the pipe internal force. However, on the other hand, it also increases the stiffness of the pipeline foundation, which results in greater vertical stress on the pipe crown, thereby increasing the pipe internal force. Due to the shallow soil cover depth, the vertical stress of the pipe crown plays a leading role. Therefore, the bending moment of the crown in Figure 17 increases as the soil elastic modulus increases. Below the spring line, the soil cover...
depth is thick and the pipe-soil interaction plays a dominant role. Therefore, the bending moment is reduced accordingly.

**Influence of different Pipe Thickness**

Figure 18 and figure 19 show the radial deformation and the circumferential bending moment when the pipe thickness is changed. The circumferential bending moment increases as the thickness of the pipe wall increases, and the radial deformation shows the opposite trend. This is because the wall thickness of the pipelines with the same diameter is different, so the bending stiffness is also different. With the thickness increasing of the pipe, the pipe section stiffness increased, resulting in the reduction of the radial deformation. And the stiffness increasing will also reduce the deformation coordination ability between the pipe and the soil and increase the vertical stress of the pipe crown, thereby increasing the stress distribution around the pipe, so the bending moment will also increase.

![Graph showing radial deformation and bending moment](image)

**Figure 16:** Radial deformation at different soil elastic modulus
Figure 17: Circumferential bending moments at different soil elastic modulus

Figure 18: Radial deformation with different pipe thickness

Figure 19: Circumferential bending moments with different pipe thickness
Conclusions

In order to analyze the stress and deformation characteristics of the underground pipeline after polymer repair, the Winkler model and the Vlazov model are be presented by considering the pipe-soil-polymer interaction based on the elastoplastic theory and the MATLAB calculation program is compiled based on the transfer matrix method. By comparing the stress and the deformation characteristics of the underground pipeline before and after polymer grouting repairing, and making use of experimental results, the accuracy and rationality of the Winkler model and the Vlazov model are discussed. Finally, the Vlazov model is chosen to study the pipe deformation characteristics after repair. The working conditions include the pipeline depth, load concentration, backfill soil elastic modulus and pipeline wall thickness, etc. The conclusions are drawn in the following.

- The radial deformation and the circumferential bending moment calculated by the Winkler model and the Vlazov model considering the pipe-soil-polymer interaction are consistent with experimental results, which verifies the accuracy and rationality of the models. The calculation results of the Vlazov model are closer to the experimental results.
- The void has a great influence on the overall mechanical properties of pipelines. The void area at the bottom significantly increases the radial deformation and the axial internal force of the pipeline. However, after the polymer grouting repair, the radial deformation and the axial internal force are close to a normal pipeline, which indicates that the polymer grouting can repair the pipeline effectively.
- With the soil cover depth increases, the radial deformation and bending moment increase significantly. This is because the overburden pressure of the pipeline will increase with the soil cover depth, resulting in an increase of the stress and deformation.
- The radial deformation of the pipeline decreases as the backfill soil elastic modulus increases. With the increase of the backfill soil elastic modulus around the pipeline, the pipe-soil interaction is also strengthened, which is beneficial
to the development of the soil medium supporting effect. It is beneficial to the stress of the entire pipe.

- The circumferential bending moment increases as the thickness of the pipe wall increases, while the trend of the radial deformation is the opposite. An increase in the thickness of the pipe wall increases the pipe section stiffness, resulting in a reduction of the pipe radial deformation.
- Polymer grouting is an effective trenchless repair technology for pipelines. It uses a self-expanding anhydrous reactive polymer material that expands and solidifies rapidly after reaction. The polymer grouting material is injected into the leaking area of the pipeline to fill the void outside the pipe and lift the settlement pipe. At present, the technology has successfully reinforced and repaired various underground pipelines. This study provides a theoretical reference for the design and construction of pipeline polymer repairs.

References

6. FM Wang, B Li, HY Fang. Experimental and numerical study on polymer grouting repair of underground pipeline with void and corrosion diseases. Hazard Control in


