Book Chapter

Thermodynamic Entropy in Quantum Statistics for Stock Market Networks

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Abstract

The stock market is a dynamical system composed of intricate relationships between financial entities, such as banks, corporations and institutions. Such a complex interactive system can be represented by the network structure. The underlying mechanism of stock exchange establishes a time-evolving network among companies and individuals, which characterise the correlations of stock prices in the time sequential trades. Here, we develop a novel technique in quantum statistics to analyse the financial market evolution. We explore the thermodynamic entropy in heat bath analogy where the normalised Laplacian matrix plays the role of the Hamiltonian operator of the network. The eigenvalues of the Hamiltonian specify energy levels of the network, which are occupied by either indistinguishable bosons or fermions obeying the Pauli exclusion principle. This provides the partition functions relevant to Bose-Einstein and Fermi-Dirac statistics. We conduct the experiments to show the thermodynamic entropy can represent network evolution and identify the significant variance in network structure during the financial crisis. This thermodynamic characterisation provides an excellent framework to represent the variations taking place in the stock market.

Introduction

The stock price is usually regarded as one of the chief representatives of economic activity in the financial market [1,2]. It reflects the interaction among each individual and company [3]. The correlation between different financial entities is a complex system that evolves with time. Exploring the dynamic evolution of such a complex system reveals the intrinsic mechanism of the financial market, and attract scientists from different fields [1-5].

To quantify such a dynamic system, tools from complex networks have been applied to study the time sequential stock market prices [1,5,15]. Generally, most available network approaches map time series into the network domain so that it presents the topological and structural properties of the system [15,23]. For example, the hierarchical structure of a minimal spanning tree provides a topological space in correlation coefficients of economic taxonomy [7]. The community structure of stock market networks represent the structural variations during the financial crisis [15].

However, most of the available work mainly focuses on the topological structure of the financial networks. They only introduce the global information of a specific period. Since the strong correlation in the time evolution of the stock market, it is significant to study the statistical properties of dynamic networks, especially during the financial crisis [4,19,20]. Recently, a robust method introduces the entropic measurement to quantify the network characterisation [6,18,19]. For example, the von Neumann entropy gives a qualitative expression for the entropy associated with the degree combinations of nodes forming edges [6,16,18].

To embark on this type of analysis, this paper is motivated by establishing effective and efficient methods for measuring the thermodynamic entropy in time-evolving networks. In particular, we analyse the stock market networks from the New York Stock Exchange (NYSE) [15]. We show that the financial crashes are characterised by the presence of well-defined changes to the thermodynamic entropy [11,16], whereas outside these critical periods this characterisation remains stable for long periods. To do this, we make use of some recent framework in quantum statistics concerning the normalised Laplacian matrix for the construction of partition functions in Bose-Einstein and Fermi-Dirac statistics [17].

Related Literature

The study of correlation of financial equities plays a vital role in improving the ability to model financial entities, such as stock portfolios and fragility. The underlying principle is the use of financial time series, from which a correlation (or covariance) matrix is estimated, to construct networks [3,14]. Then, the network characterisations shed new light on their underlying structure and dynamics.

There are different approaches to address this problem [1-5,23]. The most common one is the principal component analysis of the correlation matrix of the time sequential financial data [26]. But this method only considers the global and linear information between pairs of financial entities. More and more research finds that the intermediate connections and collective dynamics are also crucial in analysing the financial system, especially in describing the cascade effect of the crisis [14-16]. In such a case, the occurrence of extreme events is inferred from the detection of anomalies in the time series originating from the network evolution.

Recently, an investigation of the thermodynamic properties has been performed by physicists by using the perspective and theoretical results of the network theory [5,20,23]. Network entropy has been extensively used to characterise the salient features of the structure in the network dynamics [19]. For example, the von Neumann entropy can be used as an effective characterisation of network structure, commencing from a quantum analogue in which the Laplacian matrix plays the role of the density matrix [6,18]. Since the eigenvalues of the density matrix reflect the energy states of a network, this approach is closely related to the heat bath analogy in thermal physics. This provides a convenient route to use entropy to analyse network characterisations.

By mapping the heat bath analogy, the energy states of a network are captured using the eigenvalues of a matrix representation of network structure. The energy states are then populated by particles which are in thermal equilibrium with the heat bath [13,35]. As a result of this thermalisation, the energy states are occupied according to the quantum statistical distribution. By specifying the microstates of the network system, statistical thermodynamics can provide deep insights into network behaviour [17].

The thermodynamic picture offered by quantum Bose-Einstein and Fermi-Dirac statistics can be described by relevant partition functions [8-10]. Thermodynamic characterisations of the network, such as entropy, can then be derived from the partition functions [13]. Commencing from the heat bath analogy with the Laplacian matrix playing the role as the Hamiltonian, the energy states of the system are occupied according to Bose-Einstein and Fermi-Dirac statistics respectively [35]. Making different choices for the partition function, we obtain different occupation statistics for the energy levels. Then, the thermodynamic entropy can be achieved by each statistical case. In qualitative terms, the Pauli exclusion principle means that particles subject to Fermi-Dirac statistics are populating the energy states less densely than Bose-Einstein statistics, since particles obeying Bose-Einstein statistics are indistinguishable, they can aggregate in the same energy state.

The thermodynamic picture offered by quantum Bose-Einstein and Fermi-Dirac statistics also manifests itself in different ways. For example, at low temperatures where there is little thermal disruption of the occupation pattern dictated by the Pauli exclusion principle, Bosons tend to condense in the lowest energy states[8], while there is just one Fermion per energy state [9,10]. As a result, thermodynamic quantity, such as entropy of the system, samples the spectrum of Laplacian energy states in different ways, and potentially convey different aspects of network structure. For instance, at low temperature under Bose-

Einstein statistics, the particles in the heat bath are likely to respond more strongly to the spectral gap (the difference between the zero and first non-zero normalised Laplacian eigenvalues) and are thus sensitive to cluster or community structure [27]. Fermi-Dirac statistics, on the other hand, are sensitive to a larger portion of the spectrum and are more sensitive to the density of energy states. As a result, they are more sensitive to the details of the degree distribution and also to structural artifacts requiring more information concerning the Laplacian spectrum such as the path length and cycle length distributions [28,29].

Paper Outline

The aim of this paper is to explore the behaviour of the thermodynamic entropy from quantum statistics in stock market networks. In particular, we validate our framework by analysing time-evolving networks constructed through correlation coefficients between stocks traded at the New York Stock Exchange (NYSE). We show that the financial crashes are characterised by the presence of salient fluctuation in thermodynamic entropy. To do this, we make use of some recent results from spectral graph theory concerning the construction of the normalized Laplacian matrix for partition function in quantum statistics.

This paper is organized as follows. In Sec. II we specify how the time-evolving network of the financial market is constructed, and describe some basic concepts in network representation. In Sec. III we present the methodology used to derive thermodynamic entropy using the network Hamiltonian and partition function. We highlight the relevance of quantum statistics, i.e. Bose-Einstein and Fermi-Dirac statistics, for the financial market characterisation. In Sec. IV, we provide our experimental results and evaluation. Finally, in Sec. V we present the conclusions of the study.

The Time-Evolving Stock Market Networks

Stock Market Dataset

The New York Stock Exchange dataset contains the daily prices of 3,799 stocks which had been traded continuously on the New York Stock Exchange for over 6005 trading days. The stock prices were obtained from the Yahoo! financial database (http://finance.yahoo.com). A total of 347 stocks were selected from this set, all of which listed the historical stock prices from January 1986 to February 2011 [15]. For these stocks, we apply the logarithm of return *R* in Eq.(1) to describes the closure price of stocks over the trading period [1,3].

$$R_i(t) = \log P_i(t) - \log P_i(t-1) \qquad (1)$$

where $P_i(t)$ is the *i*-th stock price at day *t*. The advantage of using the logarithm of return price, instead of the stock price directly, is that it is independent of inflation and discount factors and does not require the nonlinear or stochastic transformations to correct some common trends [33,34]. Thus, the stock market dataset contains the closure prices of 347 stocks over the period of 6004 days.

Stock Market Networks

In our network representation, the nodes correspond to various stocks and the edges indicate that there is a statistical similarity between the time series associated with the stock closing prices. In particular, to determine the edge structure of the network, we apply the Pearson correlation coefficient in Eq.(2) to quantify the similarity between two time sequential stock prices.

$$\rho_{ij} = \frac{\overline{R_i R_j} - \overline{R_i}\,\overline{R_j}}{\sqrt{\left(\overline{R_i^2} - \overline{R_i}^2\right)\left(\overline{R_j^2} - \overline{R_j}^2\right)}} \qquad (2)$$

where R_i is the logarithm of return. Therefore, we obtain a fully weighted matrix of correlation coefficients which represents the weight of edges by ρ_{ij}.

However, the correlation coefficient matrix cannot straightly represent the topology structure of financial networks, since it does not fulfil the definition of axioms of a metric. In order to analyse the network structure using the adjacency matrix, we set a threshold ϵ to get a strong connection matrix for the edges. This leads to the definition of stock market networks by,

$$A_{ij} = \Theta\left(\rho_{ij} - \epsilon \right) - \delta_{ij} \qquad (3)$$

where $\Theta(\cdot)$ is the Heaviside function [21] and $\delta_{ij}$is the Kronecker delta [22].

To analyse the time evolution of the stock market networks, we use a time window to compute the correlation coefficients between the time-series for each stock pair [15]. Specifically, as shown in Figure 1, we set the length of time window $\Delta t = 30$ days insides which the network is constructed by the correlations. Connections are created between a stock pair if the correlation exceeds a determined threshold. In our experiments, we set the correlation coefficient threshold to the value to $\epsilon = 0.85$ so that $\eta = 10\%$ of all possible *N(N-1)/2* edges remained at each time. The empirical results show that there are no significant changes for the network entropy if η belongs to the range [5%, 25%]. Then, we sequentially slide the window by $\delta t = 1$ to generate a sequence of networks according to the stock market time [15]. This yields a time-varying stock market network with a fixed number of 347 nodes and varying edge structure for each of the 6,000 trading days. The edges of the network, therefore, represent how the closing prices of the stock follow each other.

Network Representation

Let *G(V,E)* be an undirected network with node set *V* and edge set $E \subseteq V \times V$, and let |*V*| represent the total number of nodes on network *G(V,E)*. The adjacency matrix *A* of a network is defined as

$$A = \begin{cases} 1, \; if \left(u,v\right) \in E \\ 0, \; otherwise \end{cases} \qquad (4)$$

Then the degree of node u is $d_u = \sum_{v \in V} A_{uv}$.

The normalised Laplacian matrix $\tilde{L}$ of the network G is defined as $\tilde{L} = D^{-1/2} L D^{1/2}$, where $L = D - A$ is the Laplacian matrix and D denotes the degree diagonal matrix whose elements are given by $D(u,u) = d_u$ and zeros elsewhere. The element-wise expression of $\tilde{L}$ is

$$\tilde{L}_{uv} = \begin{cases} 1, & if\ u = v\ and\ d_u \neq 0 \\ -\frac{1}{\sqrt{d_u d_v}}, & if\ u \neq v\ and\ (u,v) \in E \\ 0, & otherwise. \end{cases} \quad (5)$$

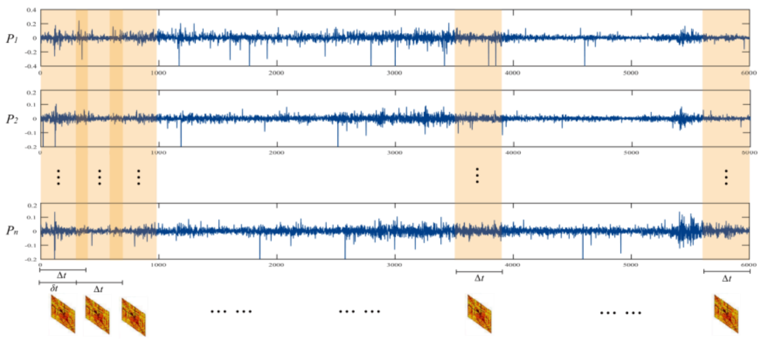


Figure 1: The illustration of the method to construct stock market networks. The network is constructed by calculating the correlations between the stocks return prices Pi (i = 1, 2, ...,N) inside a time window of length Δt. Next, by shifting this time window by amounts δt until the end of the database is reached, we obtain the network evolution.

Quantum Statistics in Networks

Quantum statistics can be combined with network theory to characterise network properties. The network can be viewed as a grand canonical ensemble, and the thermal quantities, such as energy and entropy, depending on the assumptions concerning the Hamiltonian for the system and the corresponding partition function [35].

Network Hamiltonian

In quantum mechanics, the Hamiltonian operator is the sum of the kinetic energy and potential energy of all the particles within a system [30]. It is the energy operator of a system. The standard formulation on a manifold is

$$\hat{H} = -\nabla^2 + U(r,t) \qquad (5)$$

In our case, we assume that the graph is in contact with a heat reservoir. The eigenvalues of the Laplacian matrix can be viewed as the energy eigenstates, which determine the Hamiltonian and, hence, the relevant Schrödinger equation which governs the particles in the system [11]. The particles occupy the energy states of the Hamiltonian subject to thermal agitation by the heat bath. The number of particles in each energy state is determined by the temperature, the assumed model of occupation statistics and the relevant chemical potential [13,17].

If we take the kinetic energy operator $-\nabla^2$ to be the negative of the adjacency matrix, i.e. -A, and the potential energy *U(r,t)* to be the degree matrix D, then the Hamiltonian operator is the Laplacian matrix [36]. Similarly, the normalised form of the graph Laplacian is identical to the Hamiltonian operator

$$\hat{H} = \tilde{L} \qquad (6)$$

In this case, the energy states within the network $\{\varepsilon_i\}$ are the eigenvalues of the Hamiltonian. The eigenvalues are all greater than or equal to zero, and the multiplicity of the zero eigenvalues is the number of connected components within the network.

Thermodynamic Quantities

We consider the network as a thermodynamic system specified by *N* particles with energy states given by the Hamiltonian operator, and it is immersed in a heat bath with temperature *T*. The ensemble is represented by a partition function *Z(β, N)*, where *β* is inverse of temperature [35]. When specified in this way the various thermodynamic characterisations can be computed for the networks. For instance, the average energy is given by

$$U = \left[-\frac{\partial}{\partial \beta} logZ\left(\beta, N\right) \right]_N = k_B T^2 \left[\frac{\partial}{\partial T} logZ\left(T, N\right) \right]_N \quad (7)$$

the thermodynamic entropy by

$$S = logZ + \beta U = k_B \left[\frac{\partial}{\partial T} TlogZ\left(T, N\right) \right]_N \quad (8)$$

and the chemical potential by

$$\mu = -k_B T \left[\frac{\partial}{\partial N} logZ\left(T, N\right) \right]_\beta \quad (9)$$

For each distribution, we capture the statistical mechanical properties of particles in the system using the partition function associated with the different occupation statistics [12]. The network can then be characterised using thermodynamic quantities computed from the partition function, and these include the entropy, energy and temperature.

Bose-Einstein Statistics

The Bose-Einstein distribution applies to indistinguishable bosons. Each energy state specified by the network Hamiltonian can accommodate an unlimited number of particles [8]. Bosons subject to Bose-Einstein statistics can aggregate in the same energy state because they are not subject to Pauli exclusion principle [8].

In a network, that contains a grand-canonical ensemble with a varying number of particles N and a chemical potential μ, the Bose-Einstein partition function is

$$Z_{BE} = det\left[I - exp\left(\beta\left(\mu - \tilde{L}\right)\right) \right]^{-1} = \prod_{i=1}^{V} \left(\frac{1}{1 - e^{\beta\left(\mu - \varepsilon_i\right)}} \right) \quad (10)$$

The corresponding entropy is

$$S_{BE} = logZ - \beta \frac{\partial logZ}{\partial \beta} = \sum_{i=1}^{V} log\left(1 - e^{\beta(\mu-\varepsilon_i)}\right) - \beta \sum_{i=1}^{V} \frac{(\mu-\varepsilon_i)e^{\beta(\mu-\varepsilon_i)}}{1-e^{\beta(\mu-\varepsilon_i)}} \qquad (11)$$

The entropy depends on the chemical potential for the partition function and hence it is determined by the number of particles in the system. At the temperature β, the corresponding number of particles in the level i with energy ε_i is

$$n_i = \frac{1}{exp\left[\beta\left(\varepsilon_i - \mu\right)\right] - 1} \qquad (12)$$

As a result, the total number of particles in the system is

$$\mathrm{N} = \sum_{i=1}^{V} n_i = \sum_{i=1}^{V} \frac{1}{exp[\beta(\varepsilon_i-\mu)]-1} = Tr\left[\frac{1}{exp[\beta(\tilde{L}-\mu)]-I}\right] \qquad (13)$$

In order for the number of particles in each energy state to be non-negative, the chemical potential must be less than the minimum energy level, i.e. $\mu < \min \varepsilon_i$.

Since Bose-Einstein statistics allow particles to congregate in the lower energy levels, the corresponding energy and entropy most strongly reflect the smaller Laplacian eigenvalues. As a result, the number of connected components (the multiplicity of the zero eigenvalues), and spectral gap (the degree of bi-partiality in a graph) are the most strongly reflected.

Fermi-Dirac Statistics

The Fermi-Dirac distribution applies to indistinguishable fermions with a maximum occupancy of one particle in each energy state [9,10]. Particles cannot be added to states that are already occupied and, hence, they obey the Pauli exclusion principle [9,10].

These particles behave like a set of free fermions within a complex network with energy states determined by the network Hamiltonian. The statistical properties of the networks are thus given by the Fermi-Dirac statistics of the equivalent quantum system, and the corresponding partition function is

$$Z_{FD} = det\left[I + exp\left(\beta\left(\mu - \tilde{L}\right)\right)\right] = \prod_{i=1}^{V} 1 + e^{\beta(\mu-\varepsilon_i)} \qquad (14)$$

The associated entropy of the Fermi-Dirac system is given by

$$S_{FD} = logZ - \beta \frac{\partial logZ}{\partial \beta} = \sum_{i=1}^{V} log\left(1 + e^{\beta(\mu-\varepsilon_i)}\right) - \beta \sum_{i=1}^{V} \frac{(\mu-\varepsilon_i)e^{\beta(\mu-\varepsilon_i)}}{1+e^{\beta(\mu-\varepsilon_i)}} \qquad (15)$$

In accordance with the principle of Fermi-Dirac statistics, on the other hand, the number of particles occupying the *i*th energy state is

$$n_i = \frac{1}{exp\left[\beta\left(\varepsilon_i - \mu\right)\right]+1} \qquad (16)$$

and the total number of particles in the network system is

$$\mathrm{N} = \sum_{i=1}^{V} n_i = \sum_{i=1}^{V} \frac{1}{exp[\beta(\varepsilon_i-\mu)]+1} = Tr\left[\frac{1}{exp[\beta(\tilde{L}-\mu)]+I}\right] \qquad (17)$$

With a single particle per energy state, the chemical potential is the *n*th energy level, and so $\mu = \varepsilon_n$. Since Fermi-Dirac statistics exclude multiple particles from the same energy level, the corresponding energy and entropy do not reflect the lower part of the Laplacian spectrum. Additionally, they are sensitive to a greater portion of the distribution of Laplacian eigenvalues. As a result, we might expect them to be more sensitive to subtle differences within a network structure.

Experiments and Evaluations

Experimental Results

We now explore whether the thermodynamic entropy derived from the quantum statistics can be employed as a useful tool for better understanding the evolution of networks. We first investigate whether the thermodynamic entropy can be used to characterise the changes of network structures in time series.

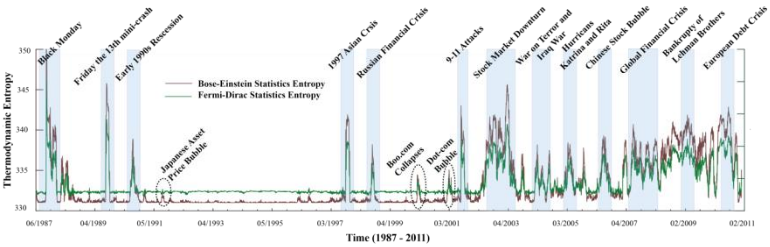


Figure 2: Entropy in NYSE (1987-2011) derived from Bose-Einstein and Fermi-Dirac statistics. Critical financial events, i.e., Black Monday, Friday the 13th mini-crash, Early 1990s Recession, 1997 Asian Crisis, 9.11 Attacks, Downturn of 2002-2003, 2007 Financial Crisis, the Bankruptcy of Lehman Brothers and the European Debt Crisis, etc. It is efficient to use thermodynamic entropy to identify critical events in NYSE.

Figure 2 shows the thermodynamic entropy for the NYSE times series data. It is annotated to indicate the positions of significant financial events such as Black Monday, Friday the 13th mini-crash, Early 1990s Recession, 1997 Asian Crisis, 9.11 Attacks, Downturn of 2002-2003, 2007 Financial Crisis, the Bankruptcy of Lehman Brothers and the European Debt Crisis [19,20]. In each case, the entropy undergoes significant fluctuations during the financial crises, associated with dramatic structural changes. A good example is the downturn of 2002-2003. After the 9.11 attacks, investors became unsure about the prospect of terrorism affecting the United States economy. Following the subsequent collapse of many internet companies, numerous large corporations were forced to restate earnings and investor confidence suffered [23]. This considerably altered the inter-relationships among stocks and resulted in significant variance in the structure of the entire market.

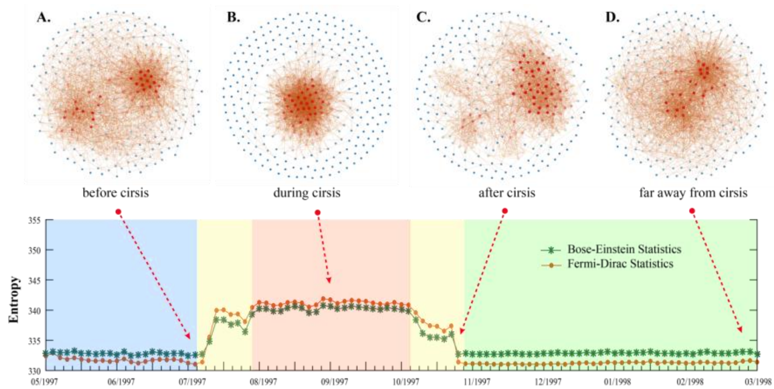


Figure 3: Thermodynamic entropy of the NYSE networks in distinct times during and around the 1997 Asian financial crisis. We show a visualisation of the network at four specific days. Node colours correspond to the degree found for the network. We note that the average degrees of networks A, B, C and D are the same.

In order to better understand the evolution of the financial market network, it is useful to visualise how its structure is organised near a critical event associated with entropy. Take the 1997 Asian financial crisis as an example. This event can be used as a reference for the effect of financial instabilities in the network structure, which occurred between July 1997 to November 1997 [23-25]. The representation of financial stability can be described by the network structure with the thermodynamic entropy. As shown in Figure 3, we note that the community structure or the connected components of the network always correspond to the fluctuation of thermodynamic entropy. In Figure 3, we show the structure visualisations associated with four different instants of time, where the node colour represents the density of degree connections. Furthermore, in order to correctly observe the thermodynamic evolution, the parameters of temperature and particle numbers are kept fixed for the four instant times in the visualisation of networks A, B, C and D.

From the figure, it is clear to view that before the crisis the network structure is mainly composed of two predominant communities and the thermodynamic entropy remain stable at the low-value area. As the network approaches the crisis, the network structure changes drastically. Only a highly connected cluster at the centre of the network remains. The two community structures substantially vanish and the value of entropy tends to climb up. During the crisis, the network structure exhibits a more homogeneous connection, as represented by the higher values of entropy. At this epoch, most stocks are disconnected, meaning that the prices evolve without strong correlations. Similar patterns of the 1997 Asian Crisis can be found in temporal network analysis. This result also agrees with other findings on the structural organisation of financial market networks [23-26]. Throughout the crisis period, the connected compositions preserve most of their communities, and the entropy becomes to decrease to the low value. After the crash in a long period, the network recovers to connected again.

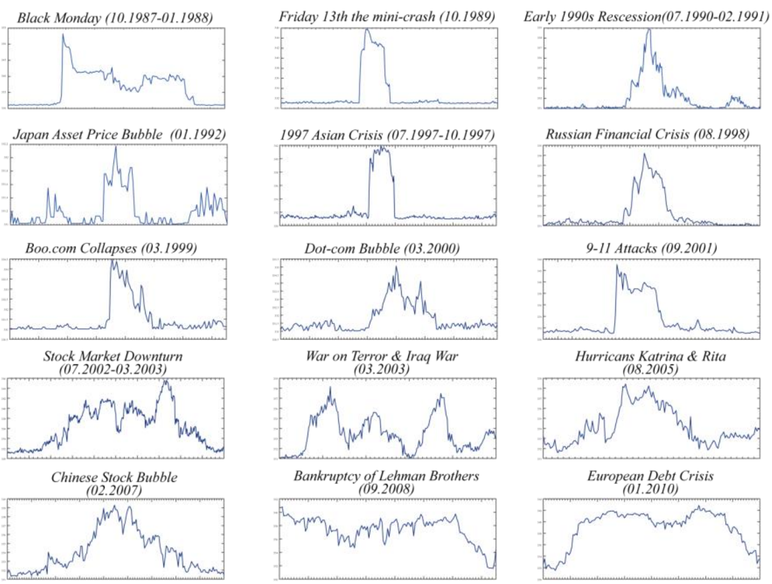


Figure 4: The individual time series of the stock market network. The thermodynamic entropy for all the different global events that have been identified.

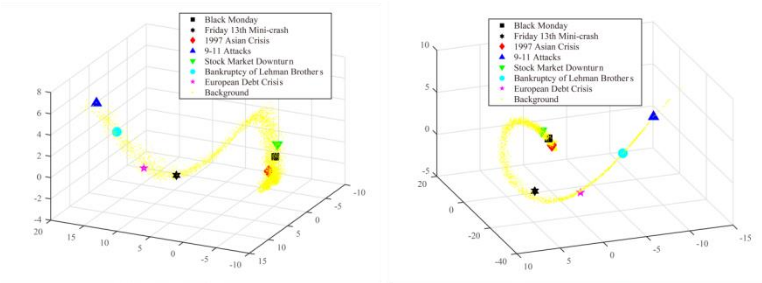
In order to quantitatively investigate the relationship between a financial crisis and thermodynamic entropy, we present a set of critical crisis periods in Figure 4. These periods are marked alongside the curve of the thermodynamic entropy in Bose-

Einstein statistics, which exhibits a similar tendency in Fermi-Dirac statistics. As shown in Figure 4, the most striking observation is that almost all of the largest peaks and troughs can find their realistic financial crisis correspondences, which show the thermodynamic entropy is sensitive to network structural changes.

In addition, for each considered crisis, we observe different detailed behaviours around the time span of the crisis. For example, both Friday 13th the mini-crash and 1997 Asian crisis present a sharp trough and peak in the corresponding time series, which dramatically change the network structure in a short time. On the other hand, Bankruptcy of Lehman Brothers and European Debt Crisis exhibit a persistent influence on the stock market with a broad entropic fluctuation in those periods. Therefore, this indicates that the thermodynamic entropy can capture network characterisations related to the financial crisis at different times.

Evaluations

We now compare our thermodynamic entropy with other thermodynamic characterisations, namely, the heat kernel signature [31] and the wave kernel signature [32], to analyse the dynamic financial networks. Figure 5 shows three-dimensional scatter plots obtained from the principal component analysis (PCA) of network representations respectively. Both plots show a compact manifold structure. However, the smooth and compact manifold trajectory does not identify the critical points, such as Black Monday, 1997 Asian Crisis and Stock Market Downturn. This indicates that although thermodynamic characterisation is effective to analyse financial network evolution, other thermal representation methods preserve information concerning significant changes in network evolution compared to the thermodynamic entropy [13,15].



(a) Heat Kernel Signature (b) Wave Kernel Signature

Figure 5: The 3D visualisation of PCA plots in the dynamic stock correlation networks described by other thermodynamic characterisation methods. (a) heat kernel signature; (b) wave kernel signature.

Finally, we focus in detail on a critical financial event, namely, the 1997 Asian crisis, to explore the dynamic structural difference with the entropic variance. We decompose the edge entropy by using the eigenvector of the Laplacian matrix and replacing its eigenvalues with the thermodynamic entropy elements. As shown before in Figure 3, the network structure has a dense cluster before the crisis and the number of connections decreases significantly during the financial crash. After that, the stocks begin to recover connections with another and a few stocks tend to form some clusters in the network structure. This phenomenon also reflects on the edge of entropy decomposition. Figure 5 shows the edge entropy distribution around the crisis for two quantum statistics. There is a narrow distribution during the 1997 Asian crisis, compared with a broader edge entropy distribution before and after the crash.

Moreover, an interesting observation is the difference of edge entropy distribution between Bose-Einstein and Fermi-Dirac statistics after the Asian Crisis. This is because the networks exist some clusters with community structure. Since Bose-Einstein statistics preferentially sample the lower energy levels with the network eigenvalue spectrum, it is more suitable to detect networks with strong community edge connection [12]. While Fermi-Dirac statistics may be more sensitive to the mean and variance of the eigenvalue distribution since they probe a wider range of energy levels [17].

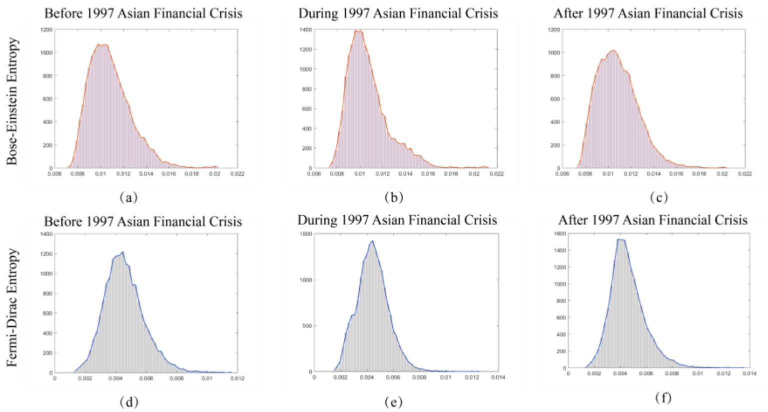


Figure 6: Edge entropy distribution of network structure before, during and after the 1997 Asian financial crisis. (a)-(e) Bose-Einstein statistics. (d)-(f) Fermi-Dirac statistics.

In conclusion, the thermodynamic entropy from quantum statistics can provide an effective tool to represent the dynamic structure of network evolution. To explore a more detail, Bose-Einstein statistics is more sensitive to reflect strong community edge connection; while Fermi-Dirac edge entropy is more suitable to represent high degree variations.

Conclusions

The study of stock market networks not only improves the decisions related to the industrial entities but also provides a reliable indicator for an imminent widespread stock value decline, which refers to a financial crisis. This description of the network evolution tends to convey the dynamic financial market which infers the underlying financial activities and partnerships.
The goal of this paper is to show that thermodynamic entropy can be used to describe the dynamics of stock market networks. Here, we explore the thermodynamic characterisations resulting from different choices of quantum statistics, i.e. Bose-Einstein statistics and Fermi-Dirac statistics, in a heat-bath analogy. The method is based on uses the normalised Laplacian matrix as the Hamiltonian operator of the network. The thermodynamic

entropy is, then, computed from the partition functions for Bose-Einstein and Fermi-Dirac energy level occupation statistics.

The results indicate that it is suitable to use the thermodynamic entropy to attest the statistical significance of experimental observations on stock market networks. Entropy in quantum statistics can provide an indicator to identify the financial crisis during the network evolution. Moreover, both entropies in Bose-Einstein and Fermi-Dirac statistics are effective in characterising dynamic network structure. But the phenomenon of Bose-Einstein and Fermi-Dirac statistics are significant different producing quite different edge entropy characterisations of network structure. Bose-Einstein system condenses into a state where the particles occupy the lowest energy state, which preferentially samples the lower energy levels with the network eigenvalue spectrum. The resulting entropy is more suitable to detect networks with strong community edge connection. Fermi-Dirac system, on the other hand, follows the Puli exclusion principle with only one particle per energy level. It probes a wider range of network spectrum which is more sensitive to the mean and variance of the eigenvalue distribution.

In addition, it also remains to be studied if different pruning techniques, used to transform a correlation matrix into the adjacency matrix, can improve the results or provide new insights about the data. In addition, other datasets related to the financial market, such as interbank ownership, could provide additional developments about the relevance of thermodynamic characterisations during pronounced market crises.

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