Book Chapter

Exact Analytical Solution for Non-Stationary Linear Inverse Heat Conduction Problem for Bodies Dimensional Geometry with the Boundary Conditions on One Surface, and Also on the Two Surfaces for a Flat Body Obtained in Closed Form Recurrent

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Annotation

In this paper we obtain exact analytical solutions for a non linear inverse heat conduction problem for bodies dimensional geometry with the boundary conditions on one surface, as well as the two surfaces for a flat body obtained in closed form recurrence. The above article recurrent form of the solutions of non-stationary linear inverse heat conduction problems for bodies with a one-dimensional geometry of the boundary conditions on one surface and on two surfaces for a flat body, - a solution in closed form with one voice that it is not always possible to explicitly.

Keywords

Thermal Conductivity; Analytical; Unsteady; Linear; A One-Dimensional; Inverse Problem; Surface; Border Conditions; Unilateral; Bilateral; Recurrent; Flat; Spherical; Cylindrical

Introduction

Application Relevance of Inverse Problems of Heat Conduction and Heat Transfer

Direct mathematical modeling predicts the thermal state of a wide range of operation modes, for example, a technical system, to analyze the influence of various factors on the behavior of the system and select the optimum thermal conditions. Application Direct methods of mathematical modeling requires an analysis of the accuracy of mathematical models. The model can be quite complex and take into account a sufficiently large number of factors. However, it is necessary to set the numerical values of all the members of the model characteristics, in particular, thermal properties of materials, the characteristics of the thermal interaction with a washing medium, and others. If the information is missing or has low accuracy, the complex mathematical model loses its dignity and does not ensure the required accuracy of forecast thermal regimes.

practical use of mathematical modeling heat transfer indicates that the possible poor accuracy in mathematical modeling, for example, of high thermal processes due to low accuracy of determining the characteristics lines using conventional techniques [19]. B such cases can be very effectively use settlement and experimental methods, which are based on the principles of identification of systems with distributed parameters, which are based on algorithms and methods for solving ill-posed inverse type heat transfer problems [19].

As is known, in direct problems is desired temperature field, which is the solution of the heat equation with known parameters of internal transfer corresponding to a known initial and boundary conditions, and in the inverse heat conduction problem initial temperature distribution and the boundary conditions are unknown to be determined functions.

Inverse Problems divided into two main types:

- The definition of internal energy transfer parameters - thermal diffusivity coefficients and, the specific heat, the light absorption coefficient and the like, which are physical characteristics of the substance;
- The determination of the conditions external energy exchange between the body and the medium, i.e., finding the boundary conditions here include calculation of surface temperature and passing it through a heat flow calculation variable heat transfer coefficient, thermal contact resistances, degrees of blackness, the angular coefficients of the irradiation position of the surface of the phase transition or degradation, the composition of unsteady power and energy balances, and the like . [nineteen].

It is clear that to obtain a solution inverse heat conduction problem is much more complicated than a straight line, but in the direct problem in measuring or realization of the given boundary conditions can be many of the experimental nature of the obstacles. The physical conditions are, for example, such that practically not always possible to install the sensor on the body surface or substantially reduced measurement accuracy due to the placement of sensors. Consequently, it is often difficult to measure the variation of a heated solid body surface temperature. It is much easier to perform a sufficiently accurate measurement of the time dependences of temperature in the interior of the heat-insulated on the body surface. Thus, there is a problem the choice between a relatively inaccurate measurements and complex analytical task.

Direct heat conduction problem when correctly set conditions has a unique solution. In the case of inverse problems of possible identity as a result of temperature fields of different nature, but equivalent in energy to external influences [5,6,19].

Temperature solid-state field does not uniquely determine the boundary conditions under which it arose. A number of energy-equivalent may vary to reflect the complex thermal processes for their effects on the system of boundary conditions.

An example is the fact that any redistribution of the heat flux density, e.g., between the convective and radiative components during their combination leads to identical thermal state of the system [19].

There are other disadvantages inherent reverse research methods unsteady state heat transfer in Engineering Systems: limiting the number of points in detail, in which the measured temperature and the heat flow; experimentally certain values of temperatures and heat flows, on which the calculations are made, contain measurement errors even when using precision instruments, since occupancy sensors in a solid body in some way violates the temperature field components; curvature of the surface, the spatial and temporal variation of heat flow in the body does not make it possible to accurately predict the direction of heat flow, or in other words, to determine the location of the sensor, which should be at the normal to the surface.

It should be noted that the inverse methods do not allow physical interpretation of complex nonstationary processes that occur in the systems.

In addition to the disadvantagesincluding the above-mentioned inverse methods have some advantages as compared to direct. The direct problem in measuring or realization of the given boundary conditions can be many of the experimental nature of the obstacles. The physical conditions in the test systems may be such that no capacity sensor at the body surface (e.g., surface coatings) or substantially reduced measurement accuracy due to the placement of the sensors, so it is often difficult to measure the variation of temperature and heat flux surfaces of solids.

In summary, we can conclude that there is a relevance in obtaining a single closed exact analytical solutions of the nonstationary linear inverse heat conduction problem for bodies dimensional geometry with boundary conditions on one surface. In this article the exact closed analytical solution of the inverse heat conduction problem is achieved in a recurrent form, ie, implicitly, as it is not always possible to explicitly [2-6].

The Decision in the Recursive form for Nonstationary Linear Inverse Heat Conduction Problem for Bodies Dimensional Geometry with the Boundary Conditions on One Surface

There is an exact solution of inverse problems of unsteady heat conduction are relatively few, and significantly less than the relevant decisions of direct non-stationary heat conduction problems. You can specify that one of the first successful attempts to solve the inverse transient heat conduction problem for flat body was first made in 1890 Y.Stefanom [1].

Subsequently, for one-dimensional transient heat conduction linear inverse problem solutions were prepared independent of one another way O.R.Burggrafom [2] and D.Lengfordom [3], assuming fame at the sensor location of transient heat flux and temperature. Exact solutions for the temperature field of a known temperatures in two different interior point method Laplace transformation were obtained M.Imberom D.Khanom and [4].

Analogous solutions for one-dimensional bodies are also given in [5] and [6], in which solutions for a non temperatures are given explicitly, a heat flux density determined differentiation temperature fields.

In the future, solutions were obtained similar problems, partly with not only theoretical but also practical character, including non-linear and one-dimensional unsteady heat conduction problem [7-19].

As partly mentioned in [2-6], making the expression for a non linear inverse heat conduction problem for bodies dimensional geometry expressly not possible in all cases, so in order to obtain the final solutions have to apply additional assumptions, such as in [2] , which uses the assumption of a thin wall.

The purpose of this paper is to obtain a solution of a non-stationary linear inverse heat conduction problem for bodies dimensional geometry with the boundary conditions on one surface of the same positions in the closed recursive form, which will have prior solutions explicitly certain advantages because they can be prepared for all of the above problems, and explicitly - is not for everyone.

The equation nonlinear non-stationary heat conduction for a one-dimensional geometry and a constant curvature of the body (in this case, considered the radial coordinate) as follows [5]:

$$\frac{1}{a}\frac{\partial t}{\partial \tau} = \nabla^2 t = r^{1-k}\frac{\partial}{\partial r}\left(r^{k-1}\frac{\partial t}{\partial r}\right) = \frac{\partial^2 t}{\partial r^2} + \frac{k-1}{r}\frac{\partial t}{\partial r}, \quad (1)$$

where the k - number of final measurements: 1 - flat field; 2 - cylindrical; 3 - spherical; t - temperature; r - radial coordinate; a - thermal diffusivity.

The domain of the differential equation (1) is comprised from 0 to r_2 (Radial coordinate outer surface) of the coordinate (in the case of hollow bodies from r1 (radial coordinate inner surface) to r2) and from 0 to the actual value τ time (τ > 0).

In this dimensionless form equation can be written as follows [5]:

$$\frac{\partial T}{\partial \mathrm{Fo}} = \frac{\partial^2 T}{\partial \rho^2} + \frac{k-1}{\rho}\frac{\partial T}{\partial \rho} \quad (2)$$

Where in - Fourier criterion; T - dimensionless temperature; ρ = r / r1 - dimensionless coordinate; r1 is the radial coordinate, at which the boundary conditions are given.$\mathrm{Fo} = \frac{a\tau}{r_1^2}$

Inverse heat conduction problem for equation (1) or (2) is to find the boundary conditions on the surface of the body under certain dimensional unsteady temperature and heat flow and thermal characteristics of the material of the body does not depend on temperature.

In this article we study the thermal conductivity of the process at the moment, quite remote from the initial point in time, so the influence of the initial conditions almost no effect on the temperature distribution at the time of measurement or observation (the so-called "problem without initial conditions"). In practice this can mean a sectional view, [5,6] that if a sufficient distance from the start time aftereffect component that takes into account the effect of initial conditions become so small that it will have less measurement error of the sensors that measure the temperature and heat flow.

The component feedback-dimensional temperature field layer which is heated at the inner surface is treated by using the dimensionless coordinates, to which the heated surface corresponds to a single value (complex homochronicity relates to this inner radial coordinate) can be represented as follows [5]:

$$T(\rho, \mathrm{Fo}) = \sum_{n=0}^{\infty} T_1^{(n)}(\mathrm{Fo})\, P_{n,1} + \sum_{n=0}^{\infty} \mathrm{Ki}^{(n)}(\mathrm{Fo})\, P_{n,2} =$$

$$= \sum_{n=0}^{\infty} \Theta_{n,1}\, P_{n,1}$$

$$+ \sum_{n=0}^{\infty} \Theta_{n,2}\, P_{n,2}\ , \qquad (3)$$

wherein - Kirpichev criterion; - Fourier criterion; ρ = r / r1 - dimensionless coordinate; r1 is the radial coordinate, at which the boundary conditions are given; a - thermal diffusivity; λ - thermal conduction coefficient; q - heat flux density; Δt - temperature difference.$\mathrm{Ki} = \frac{q r_1}{\lambda \Delta t}$ $\mathrm{Fo} = \frac{a\tau}{r_1^2}$

On a heated surface of the case the boundary condition of the second kind. In this case, the heat flux density and the temperature are measured on the same surface.

Solutions for bodies of simple configuration will be different values of radial quasipolynomials Pn 1 and Pn 2.

In this work, these quasipolynomials will be resolved recurrent forms, in contrast to [2-6] and [7-19].

Flat Plate

Quasipolynomials Pn 1 and Pn 2 are as follows for the flat plate:

$$P_{n+1,1} = \int_0^{\rho} \int_0^{\rho} P_{n,1}\, d\rho d\rho; \text{ (four)}$$

$$P_{n+1,2} = \int_0^{\rho} \int_0^{\rho} P_{n,2}\, d\rho d\rho; \text{ (five)}$$

$$P_{0,1} = 1; P_{0,2} = \rho. \text{ (6)}$$

for the first quasipolynomials R1,1 and R2,1 etc., R1,2 and R2,2 etc. can be written for a flat plate:

$$P_{1,1} = \frac{\rho^2}{2}; P_{2,1} = \frac{\rho^4}{24};\ P_{3,1} = \frac{\rho^6}{720}; \dots . (7)$$

$$P_{1,2} = \frac{\rho^3}{6}; P_{2,2} = \frac{\rho^5}{120}; P_{3,2} = \frac{\rho^7}{5040}; \dots . \text{(eight)}$$

Hence, using the method of mathematical induction, we can zapisat quasipolynomials for solving the inverse problem of non-stationary heat conduction by setting the boundary conditions on the same surface of the flat plate in the recurrent form:

$$P_{n,1} = \frac{\rho^2}{2n\cdot(2n-1)} P_{n-1,1}; (9)$$

$$P_{n,2} = \frac{\rho^2}{2n\cdot(2n+1)} P_{n-1,2}. \text{(ten)}$$

Solid Cylinders

Quasipolynomials Pn, 1 for a solid cylinder will be as follows:

$$P_{n+1,1} = \int_0^\rho \frac{1}{\rho} \int_0^\rho \rho P_{n,1}\, d\rho d\rho; \text{(eleven)}$$

$$P_{0,1} = 1. (12)$$

For the first quasipolynomials R1,1 and R2,1 etc. can be written to the solid cylinder:

$$P_{1,1} = \frac{\rho^2}{4}; P_{2,1} = \frac{\rho^4}{64}; P_{3,1} = \frac{\rho^4}{2304}; \dots . (13)$$

Hence, using the method of mathematical induction, we can zapisat quasipolynomials for solving the inverse problem of non-stationary heat conduction by setting the boundary conditions on the axis of the cylinder in cploshnogo recursive form:

$$P_{n,1} = \frac{\rho^2}{4n^2} P_{n-1,1}. \quad (14)$$

The Hollow Cylinder

Quasipolynomials Pn 1 and Pn 2 are as follows for the hollow cylinder:

$$P_{n+1,1} = \int_1^\rho \frac{1}{\rho} \int_1^\rho \rho P_{n,1}\, d\rho d\rho; \text{(15)}$$

$$P_{n+1,2} = \int_1^\rho \frac{1}{\rho} \int_1^\rho \rho P_{n,2}\, d\rho d\rho; \text{(sixteen)}$$

$$P_{0,1} = 1;\ P_{0,2} = \ln\rho. \text{(17)}$$

For the first quasipolynomials R1,1 and R2,1 etc. can be written to the hollow cylinder:

$$P_{1,1} = \frac{1}{4}\rho^2 - \frac{1}{2}\ln\rho - \frac{1}{4} = \frac{1}{4}\rho^2 - \frac{1}{2}P_{0,2} - \frac{1}{4}P_{0,1}; \text{(18)}$$

$$P_{1,2} = \frac{1}{4}\rho^2\ln\rho - \frac{1}{4}\rho^2 + \frac{1}{4}\ln\rho + \frac{1}{4} = \frac{1}{4}\rho^2(\ln\rho - 1) + \frac{1}{4}P_{0,2} + \frac{1}{4}P_{0,1}; \text{(nineteen)}$$

$$P_{2,1} = \frac{1}{64}\rho^4 - \frac{1}{8}\rho^2\ln\rho + \frac{1}{16}\rho^2 - \frac{1}{16}\ln\rho - \frac{5}{64} = \\ = \frac{1}{64}\rho^4 - \frac{1}{2}P_{1,2} - \frac{1}{4}P_{1,1} - \frac{1}{16}P_{0,2} - \frac{1}{64}P_{0,1}; \text{(20)}$$

$$P_{2,2} = \frac{1}{64}\rho^4\ln\rho - \frac{3}{128}\rho^4 + \frac{1}{16}\rho^2\ln\rho + \frac{1}{64}\ln\rho + \frac{3}{128} = \\ = \frac{1}{64}\rho^4\left(\ln\rho - \frac{3}{2}\right) + \frac{1}{4}P_{1,1} + \frac{3}{128}P_{0,1} + \frac{1}{4}P_{1,2} + \frac{5}{64}P_{0,2}; \text{(21)}$$

$$P_{3,1} = \frac{1}{2304}\rho^6 - \frac{1}{128}\rho^4\ln\rho + \frac{1}{128}\rho^4 - \frac{1}{64}\rho^2\ln\rho - \frac{1}{256}\rho^2 - \frac{1}{384}\ln\rho - \frac{5}{1152} = \\ = \frac{1}{2304}\rho^6 - \frac{1}{4}P_{2,1} - \frac{1}{64}P_{1,1} - \frac{1}{2304}P_{0,1} - \frac{1}{2}P_{2,2} - \frac{1}{16}P_{1,2} - \frac{1}{384}P_{0,2}; \ldots; \text{(22)}$$

$$P_{3,2} = \frac{1}{2304}\rho^6\ln\rho - \frac{11}{13824}\rho^6 + \frac{1}{256}\rho^4\ln\rho - \frac{1}{512}\rho^4 +$$

$$\frac{1}{256}\rho^2\ln\rho+\frac{1}{512}\rho^2++\frac{1}{2304}\ln\rho+\frac{11}{13824}=(23)$$

$$=\frac{1}{2304}\rho^6\left(\ln\rho-\frac{11}{6}\right)+\frac{1}{4}P_{2,1}+\frac{3}{128}P_{1,1}+\frac{11}{13824}P_{0,1}+\frac{1}{4}P_{2,2}+\frac{5}{64}P_{1,2}+\frac{5}{1152}P_{0,2};\ \dots.$$

Hence, using the method of mathematical induction, it is possible quasipolynomials record for solving inverse transient heat conduction problem with the boundary conditions on the internal surface of the hollow cylinder in the recurrent form:

$$P_{n,1}=\frac{1}{((2n)!!)^2}\rho^2-\sum_{m=0}^{n-1}\frac{1}{\left((2(n-m))!!\right)^2}P_{m,1}-\sum_{m=0}^{n-1}\frac{2(n-m)}{\left((2(n-m))!!\right)^2}P_{m,2};\ (24)$$

$$P_{n,2}=\left(\ln\rho-\sum_{m=1}^{n}m^{-1}\right)\rho^2+\sum_{m=0}^{n-1}\frac{1}{\left((2(n-m))!!\right)^2}\sum_{l=1}^{n-m}l^{-1}P_{m,1}++\sum_{m=0}^{n-1}\frac{1}{\left((2(n-m))!!\right)^2}P_{m,2}+\sum_{m=0}^{n-1}\frac{2(n-m)}{\left((2(n-m))!!\right)^2}\sum_{l=1}^{n-m-1}l^{-1}P_{m,2}. \quad (25)$$

Solid Ball

Quasipolynomials Pn, 1 to cploshnogo ball will be as follows:
$P_{n+1,1}=\int_0^\rho\frac{1}{\rho^2}\int_0^\rho\rho^2P_{n,1}\,d\rho d\rho$; (26)

$$P_{0,1}=1\ .\ (27)$$

For the first quasipolynomials R1,1 etc. for a solid sphere can be written:

$$P_{1,1} = \frac{\rho^2}{6}; P_{2,1} = \frac{\rho^4}{120};\ P_{3,1} = \frac{\rho^4}{5040} \ldots. \text{(28)}$$

Hence, using the method of mathematical induction, we can zapisat quasipolynomials for solving inverse transient heat conduction problem by setting a boundary condition in the center of the solid sphere in a recurrent form:

$$P_{n,1} = \frac{\rho^2}{2n\cdot(2n+1)} P_{n-1,1}. \text{(29)}$$

Hollow Ball

Quasipolynomials Pn 1 and Pn 2 for a hollow sphere would be as follows:

$$P_{n+1,1} = \int_1^\rho \frac{1}{\rho^2} \int_1^\rho \rho^2 P_{n,1}\, d\rho d\rho; \text{(thirty)}$$

$$P_{n+1,2} = \int_1^\rho \frac{1}{\rho^2} \int_1^\rho \rho^2 P_{n,2}\, d\rho d\rho; \text{(31)}$$

$$P_{0,1} = 1;\ P_{0,2} = 1 - \frac{1}{\rho}. \text{(32)}$$

For the first quasipolynomials R1,1 and R2,1 etc. for the hollow sphere can be written:

$$P_{1,1} = \frac{1}{6}\frac{1}{\rho}(\rho - 1)^2(\rho + 2);\ P_{2,1} = \frac{1}{120}\frac{1}{\rho}(\rho - 1)^4(\rho + 4); \text{(33)}$$

$$P_{3,1} = \frac{1}{5040}\frac{1}{\rho}(\rho - 1)^6(\rho + 6); \ldots. \text{(34)}$$

$$P_{1,2} = \frac{1}{6}\frac{1}{\rho}(\rho - 1)^3; P_{2,2} = \frac{1}{120}\frac{1}{\rho}(\rho - 1)^5; P_{3,2} = \frac{1}{5040}\frac{1}{\rho}(\rho - 1)^7; \ldots. \text{(35)}$$

Hence, using the method of mathematical induction, it is possible quasipolynomials record for solving inverse transient heat conduction problem with the boundary conditions on the inner surface of a hollow sphere in a recurrent form:

$$P_{n,1} = \frac{1}{2n \cdot (2n+1)} (\rho - 1)^2 \frac{(\rho+2n)}{(\rho+2(n-1))} P_{n-1,1}. \ (36)$$

$$P_{n,2} = \frac{1}{2n \cdot (2n+1)} (\rho - 1)^2 P_{n-1,2}. \ (37)$$

For a given stationary boundary conditions $\Theta_{n,\,1}$ and Θn, 2 recurrence relations are as follows:

$$\Theta_{n,i} = \frac{r_1^2}{a} \frac{\partial \Theta_{n-1,i}}{\partial \tau}, \forall\, i = 1{,}2. \ (38)$$

The above ratios express a recursive form exact solution of the inverse heat conduction problem nonstationary for bodies dimensional geometry with unsteady boundary conditions defined on one side.

Recurrent form of the solutions makes it possible to address this problem with a common position in the closed form solutions of the expression as explicitly such as in [7-19], is not possible in all cases to that described in [2,5,6] .

Questions to the correctness of the inverse heat conduction problem (i.e., the existence of solutions, its uniqueness and stability) are examined in detail in [5,6], however, in this study there is no need of re-examination.

The above obtained solutions article nonstationary inverse heat conduction problem for a one-dimensional bodies have been successfully applied in a practical manner as part of the problem for conjugated determinancy maximum impact soot layer on the surface of the combustion chamber to the working fluid transient parameters in radiation-convective heat transfer [20-22], and also the design theory of heat transfer in the heat insulating package to stabilize temperature storage of perishable products [23-24].

For the conditions of the heat transfer characteristic of [23-24], Calculations of relationships generated in this article were conducted. At the same temperature boundary condition is the greatest deviation for the flat body, and the least - for solid sphere; for solid cylinder will take place intermediate value. As for the hollow cylinder and the hollow sphere to the temperature deviation is greater than the solid sphere and cylinder, respectively. Comparison of the hollow cylinder with the hollow globe shows that for small values of r2 / r1 deviation of the hollow cylinder will be less than for a hollow sphere, but for high values of r2 / r1 deviation of the hollow cylinder has to be larger than for a hollow sphere. For these conditions [23-24] above fracture occurs at the value r2 / r1 ≈ 3 2/15.

Solutions in Recursive Form for Nonstationary Linear Inverse Heat Conduction Problem for Bodies with Dimensional Geometry Boundary Temperature Conditions on Both Surfaces

Temperature fields hollow cylinder and a sphere, a plate, the edges of which are in different environments, are asymmetric, but one-dimensional. Asymmetric temperature field is obtained by measuring temperatures at the boundaries of the body, which must be previously known functions of time.

The component feedback-dimensional temperature field layer, on whose boundaries occur unsteady temperature limits are considered when using the dimensionless coordinates: the first point is taken as the origin, and the second has a single abscissa (for flat field); the first point has unit abscissa, and the second point is ρ2 (for spherical and cylindrical fields); can be represented as follows [5]:

$$T(\rho,\mathrm{Fo}) = \sum_{n=0}^{\infty} T_1^{(n)}(\mathrm{Fo})\, P_{n,1} + \sum_{n=0}^{\infty} T_2^{(n)}(\mathrm{Fo})\, P_{n,2} = \sum_{n=0}^{\infty} \Theta_{n,1}\, P_{n,1} + \sum_{n=0}^{\infty} \Theta_{n,2}\, P_{n,2}\,. \qquad (39)$$

On both surfaces there is a boundary condition of the first kind. In this case, the temperature measured at the boundary surfaces. Solutions for bodies of simple configuration will be different values of radial quasipolynomials Pn 1 and Pn 2.

In this work, these quasipolynomials will be resolved recurrent forms, in contrast to [2-6] and [7-19].

Flat Plate

Quasipolynomials Pn 1 and Pn 2 are as follows for the flat plate:

$$P_{n+1,1} = \int_0^{\rho}\int_0^{\rho} P_{n,1}\, d\rho d\rho - \rho \int_0^{1}\int_0^{\rho} P_{n,1}\, d\rho d\rho; \ (40)$$

$$P_{n+1,2} = \int_0^{\rho}\int_0^{\rho} P_{n,2}\, d\rho d\rho - \rho \int_0^{1}\int_0^{\rho} P_{n,2}\, d\rho d\rho; \ (41)$$

$$P_{0,1} = 1 - \rho; P_{0,2} = \rho. \ (42)$$

For the first quasipolynomials R1,1 and R1,2 etc. can be written for a flat plate:

$$P_{1,1} = -\frac{1}{6}\rho^3 + \frac{1}{2}\rho^2 - \frac{1}{3}\rho; \ (43)$$

$$P_{2,1} = -\frac{1}{120}\rho^5 + \frac{1}{24}\rho^4 - \frac{1}{18}\rho^3 + \frac{1}{45}\rho; \ (44)$$

$$P_{3,1} = -\frac{1}{5040}\rho^7 + \frac{1}{720}\rho^6 - \frac{1}{360}\rho^5 + \frac{1}{270}\rho^3 - \frac{2}{945}\rho; \ (45)$$

$$P_{4,1} = -\frac{1}{362880}\rho^9 + \frac{1}{40320}\rho^8 - \frac{1}{15120}\rho^7 + \frac{1}{5400}\rho^5 - \frac{1}{2835}\rho^3 + \frac{1}{4725}\rho; \dots; \ (46)$$

$$P_{1,2} = \frac{1}{6}\rho^3 - \frac{1}{6}\rho; \ (47)$$

$$P_{2,2} = \frac{1}{120}\rho^5 - \frac{1}{36}\rho^3 + \frac{7}{360}\rho; \ (48)$$

$$P_{3,2} = \frac{1}{5040}\rho^7 - \frac{1}{720}\rho^5 + \frac{7}{2160}\rho^3 - \frac{31}{15120}\rho; \ (49)$$

$$P_{4,2} = \frac{1}{362880}\rho^9 - \frac{1}{30240}\rho^7 + \frac{31}{43200}\rho^5 - \frac{31}{90720}\rho^3 + \frac{127}{604800}\rho; \ldots. \quad (50)$$

Hence, using the method of mathematical induction, we can zapisat quasipolynomials for solving the inverse problem of non-stationary heat conduction by setting the temperature boundary conditions on both boundary surfaces for a flat plate in a recursive form:

$$P_{n,1} = P_{n-1,1} - \frac{1}{(2n+1)!}\rho^{2n+1} + \frac{1}{(2n)!}\rho^{2n} + \sum_{k=0}^{2n-1}\frac{2^{2n+1-k}}{k!\,(2n-1-k)!}\left(\frac{B_{2n-1-k}}{4} - \frac{1}{(2n-k)}\frac{1}{(2n+1-k)}B_{2n+1-k}\right)\rho^k, \quad (51)$$

$$P_{n,2} = P_{n-1,2} - \sum_{k=0}^{2n+1}\frac{(-1)^{4n+2-k}}{k!\,(2n+1-k)!}\rho^k + \sum_{k=0}^{2n}\frac{(-1)^{4n-k}}{k!\,(2n-k)!}\rho^k + \sum_{k=0}^{2n-1}\sum_{l=0}^{k}\frac{(-1)^{4k-l}}{l!\,(k-l)!}\frac{2^{2n+1-k}}{(2n-1-k)!}\left(\frac{B_{2n-1-k}}{4} - \frac{1}{(2n-k)}\frac{1}{(2n+1-k)}B_{2n+1-k}\right)\rho^l, \quad (52)$$

wherein Bn - Bernoulli numbers: (- binomial coefficient, the number of combinations of N by K) [25]. For example, the first few Bernoulli numbers are: B0 = 1; B1 = -1 / 2; B2 = 1/6; B3 = 0; B4 = -1 / 30; B5 = 0; B6 = 1/42; B7 = 0; B8 = -1 / 30; B9 = 0; B10 = 5/66; B11 = 0; B12 = -691 / 2730; B13 = 0; B14 = 7/6; B15 = 0; B16 = -3617 / 510; B17 = 0; B18 = 43867/798; B19 = 0; B20 = -174611/330 ...$B_n = \frac{-1}{n+1}\sum_{k=1}^{n} C_{k+1}^{n+1} B_{n-1}, n \in \mathbb{N} C_N^K = \frac{N!}{K!(N-K)!}$

For the last quasi-Pn, 2 possible to regroup and write it in the following way:

$$P_{n,2} = P_{n-1,2} - \frac{1}{(2n+1)!}(1-\rho)^{2n+1} + \frac{1}{(2n)!}(1-\rho)^{2n} + \sum_{k=0}^{2n-1} \frac{2^{2n+1-k}}{k!\,(2n-1-k)!}\left(\frac{B_{2n-1-k}}{4} - \frac{1}{(2n-k)}\frac{1}{(2n+1-k)}B_{2n+1-k}\right)(1-\rho)^{k}. \quad (53)$$

Conclusions

- Background of solving the inverse problem of the linear one-dimensional transient heat conduction geometric forms obtained in this work in a closed recursive form, is that there is a possibility of sufficient accuracy to recover the boundary conditions on heat flow measurement sensor.
- In this paper we obtain exact analytical solutions for a non linear inverse heat conduction problem for bodies dimensional geometry with the boundary conditions on one surface, and also on the two surfaces for a flat body, obtained in recursive form.
- The resulting article recursive form of the linear feedback solutions unsteady thermal conductivity for bodies with dimensional geometry boundary conditions on one surface, and also on the two surfaces of the planar body is a solution in closed form a common position that is not always possible to explicitly .
- From a practical viewpoint solutions obtained can be used in calculating the transient temperature field and heat flux density for different materials used in aeronautical and aerospace engineering, from the measured time-dependent boundary conditions on one side and on the two surfaces for flat body.

References

1. Stefan J. Über die Theorie dir Eisbilding, insbesondere über die Eisbilding im Polarmeere. Sitzungsberichte der Kaiserlichen Akademie Wiss. Math. Germany: naturwiss. 1890; 98: 956-973.
2. Burgrave OR The exact solution of the inverse problem in the theory of heat conduction and its applications. Proceedings of the American Society of Mechanical Engineers Series C. Heat. 1964; 94-106.
3. Langford D. New analytical Solutions of the One-Dimensional Heat Equation for Temperature are Heat Flow Rate Both Prescribed at the Same Fixed Boundary (with applications to the phase change problem). Q. App. Math. – 1976; 24: 315-322.
4. M Imbert, Khan D. Calculation of non-stationary temperature distribution Based on indications tepmopar disposed inside the body. Missile technology and astronautics. 1972; 83-90.
5. AG Temkin Inverse heat conduction problems. Moscow: Energia. 1973; 464.
6. J Beck, Blackwell B, Saint Clair Ch. Incorrect inverse heat conduction problems. Moscow: Mir. 1989; 312.
7. Kavtaradze RZ, Lapushkin NA, Lobanov IE. Investigation of the action of the heat insulating layer of coke on the surfaces of the COP using diesel inverse heat conduction methods and conjugate. Math. universities. Engineering. 1997; 66-71.
8. Kavtaradze RZ, Lapushkin NA, Lobanov IE. Calculation-experimental study of unsteady heat transfer in the combustion chamber speed diesel engine with the action of the heat insulating layer of coke. Improvement cardinality, economic and environmental performance of the engine. Proceedings of the VI International Scientific and Practical Workshop. Vladimir. 1997; 111-112.
9. Kavtaradze RZ, Lapushkin NA, Lobanov IE. Investigation of the action of the heat insulating layer of varnish using the inverse heat conduction methods and conjugate. Motor 97. Proceedings of the international scientific and technical conference. 1997; 25.

10. Lobanov IE Analytical solution of nonlinear inverse heat conduction problem for a body with a low coefficient of thermal conductivity of one-dimensional geometry. Problems of gas dynamics and heat and mass transfer in power plants: Program XII School-Seminar of Young Scientists and Specialists under the leadership of RAS academician A.I.Leonteva. Moscow: MEI. 1999; 10.
11. Lobanov IE. Exact analytical solutions of nonlinear nonstationary inverse heat conduction problem for a body with a low thermal conductivity. Proceedings of the universities. Aviation equipment. 2010; 72-74.
12. Lobanov IE. Contact one-dimensional nonlinear heat conduction problem: exact analytical solutions. Electronic scientific journal Thermophysics and Heat Engineering. 2012; 3-12.
13. Lobanov IE. Exact analytical solutions of nonlinear nonstationary inverse heat conduction problem for bodies with a low coefficient of thermal conductivity of one-dimensional geometry. Almanac of Modern Science and Education. Tambov: Charter. 2010; 56-64.
14. Lobanov IE. Nonlinear transient inverse heat conduction problem for body-dimensional geometry with a low coefficient of thermal conductivity: exact analytical solutions. Thermal processes in the art. 2012; 4: 274-283.
15. Lobanov IE. Exact analytical solutions of nonlinear nonstationary inverse heat conduction problem for bodies with a low coefficient of thermal conductivity of one-dimensional geometry. Proceedings of the XIV Minsk International Forum on Heat and Mass Transfer. – Minsk. Section number 7. General Issues heat and mass transfer and thermal conductivity. - Report number 1-19. 2012; 1-11.
16. Lobanov IE. Exact analytical solutions of nonlinear nonstationary inverse heat conduction problem for bodies with a low coefficient of thermal conductivity of one-dimensional geometry. Theses of reports and messages XIV. Minsk: Minsk International Forum on Heat and Mass Transfer. 2012; 729-732.
17. Lobanov IE. Theoretical and experimental determinancy

unsteady state temperature varnish layer to the combustion thermal engine E. chambers periodic peer-reviewed scientific "SCI-ARTICLE.RU" magazine. 2016; 194-206.

18. Lobanov IE. On the question of determinacy influence copper film in the surface temperature of the sensor assembly in the calculation of the thermal state of fouling layer. Electronic periodical peer-reviewed scientific journal "SCI-ARTICLE.RU". 2017; 142-148.
19. Lobanov IE, Paramonov NV. Measurement and Modeling of thermal loads in the chambers of internal combustion engines. Moscow: MAI Publishing House. 2012; 160.
20. Lobanov IE. Dotsenko AI Influence of the layer of coke on the surfaces of the combustion chambers on the parameters of the working fluid. Mechanization construction. 2009; 23-26.
21. Lobanov IE. Theoretical determination maximum impact soot layer on the surface of the combustion chamber to the working fluid transient parameters in radiation-convective heat transfer. Moskovskoe scientific review. 2013; 11-15.
22. Lobanov IE. Settlement and experimental method of indirect measurement of the thickness of the layer of coke on the surfaces of heat engines combustion chambers. Electronic periodical peer-reviewed scientific magazine "SCI-ARTICLE.RU". 2016; 96-100.
23. Belozyorov GA, Lobanov IE. The use of thermal insulation package for the stabilization of temperature modes of storage of perishable products. Actual problems of modern science. 2012; 193-200.
24. Lobanov IE. The theory of heat insulating package to stabilize the temperature modes of storage of perishable products. Electronic scientific journal Studies of technical sciences. 2011; 1: 3-10.
25. H Bateman, A Erdelyi. Higher transcendental functions. Hypergeometric function. Legendre functions. Home edition of Physical and Mathematical Literature. Dongcheng: Science Publishing House. 1965; 296.